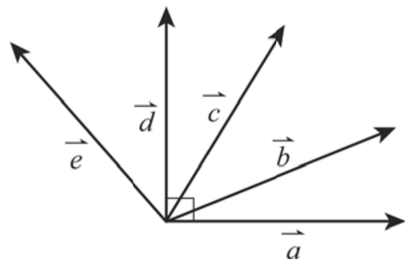
高雄市明誠中學 高二數學平時測驗 日期：99.09.16					
範圍	1-1.2 向量的基本應用	班級	二年____班	姓名	
		座號			

一、選擇題 （ 每題分 ）

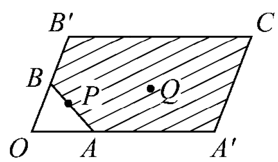


（　　　）1.下圖為五個等長的向量，試問向量 $\vec{a}$ 與下列哪一個向量的內積最小？

(1) $\vec{a}$　(2) $\vec{b}$　(3) $\vec{c}$　(4) $\vec{d}$　(5) $\vec{e}$.

解答　5

解析　內積 $\vec{a}\cdot\vec{b}$ 即 $\vec{a}$ 的長度 $|\vec{a}|$ 與 $\vec{b}$ 在 $\vec{a}$ 方向上的投影量之乘積，又 $\vec{a}$，$\vec{b}$，$\vec{c}$，$\vec{d}$ 與 $\vec{e}$ 在 $\vec{a}$ 方向上的 5 個投影量中，以 $\vec{e}$ 的投影量為負最小，$\vec{a}\cdot\vec{e}$ 最小 . 故選(5) .



（　　　）2.(多選)如圖示，A' 與 B' 分別為射線 $\overrightarrow{OA}$ 及 $\overrightarrow{OB}$ 上的點，$\overline{OA'}=3\overline{OA}$，$\overline{OB'}=2\overline{OB}$，今作平行四邊形 $OA'CB'$. 已知 P 為線段 $\overline{AB}$ 上的一點，而 Q 為斜線區域內的一點，設 $\overrightarrow{OP}=x\overrightarrow{OA}+y\overrightarrow{OB}$，

$\overrightarrow{OQ}=r\overrightarrow{OA}+s\overrightarrow{OB}$，則下列敘述何者為真？

(1) $x\ge 0$，$y\ge 0$　(2) $x+y=1$　(3) $0\le r\le 3$　(4) $1\le s\le 2$　(5) $r+s\ge 1$.

解答　1235

解析　$\overrightarrow{OP}=x\overrightarrow{OA}+y\overrightarrow{OB}\Rightarrow x+y=1$，$x\ge 0$，$y\ge 0$，

$\overrightarrow{OQ}=r\overrightarrow{OA}+s\overrightarrow{OB}$，則斜線區域為 $r+s\ge 1$ 且 $0\le r\le 3$ 且 $0\le s\le 2$，

當 $r=\frac{3}{2}$，$s=0$，即 $\overrightarrow{OQ}=\frac{3}{2}\overrightarrow{OA}$ 落在斜線區域內，但 $s=0$，$\therefore$(4)不正確

故選(1)(2)(3)(5) .

二、填充題 （ 每題 10 分 ）

1. 設 $\vec{a}+2\vec{b}+\vec{c}=\vec{0}$ 且 $|\vec{a}|=4$，$|\vec{b}|=2$，若 $\vec{a}$，$\vec{b}$ 之夾角為 $60°$，則 $|\vec{c}|=$____________ .

解答　$4\sqrt{3}$

解析　$\because\ \vec{a}+2\vec{b}+\vec{c}=\vec{0}$，$\therefore\ \vec{c}=-\vec{a}-2\vec{b}$

$$\left|\overrightarrow{c}\right|^2=\left|-\overrightarrow{a}-2\overrightarrow{b}\right|^2=\left|\overrightarrow{a}\right|^2+4\left|\overrightarrow{b}\right|^2+4\left|\overrightarrow{a}\right|\cdot\left|\overrightarrow{b}\right|\cdot\cos 60^\circ \quad =16+16+4\cdot4\cdot2\cdot\frac{1}{2}=48,$$

$$\therefore\left|\overrightarrow{c}\right|=\sqrt{48}=4\sqrt{3}\ .$$

2. 已知$\left|\overrightarrow{a}\right|=4$，$\left|\overrightarrow{b}\right|=3$，$\overrightarrow{a}$ 、$\overrightarrow{b}$ 的夾角為60°，求(1)$\left|\overrightarrow{a}+t\overrightarrow{b}\right|$的最小值為_____；(2)此時$t=$_____.

解答 (1) $2\sqrt{3}$;(2) $-\frac{2}{3}$

解析 $\left|\overrightarrow{a}+t\overrightarrow{b}\right|^2=\left(\overrightarrow{a}+t\overrightarrow{b}\right)\cdot\left(\overrightarrow{a}+t\overrightarrow{b}\right)=\left|\overrightarrow{a}\right|^2+2t\,\overrightarrow{a}\cdot\overrightarrow{b}+t^2\left|\overrightarrow{b}\right|^2$

$$=4^2+2\cdot4\cdot3\cdot\cos 60^\circ\cdot t+3^2\cdot t^2=9t^2+12t+16 \quad =9\left(t+\frac{2}{3}\right)^2+12$$

當$t=-\frac{2}{3}$時，可得最小值為$2\sqrt{3}$.

3. $\left|\overrightarrow{u}\right|=3$，$\left|\overrightarrow{v}\right|=4$，$\left|2\overrightarrow{u}+\overrightarrow{v}\right|=2\sqrt{7}$，則$\overrightarrow{u}$ ，$\overrightarrow{v}$ 之夾角為____________ .

解答 120°

解析 $\left|2\overrightarrow{u}+\overrightarrow{v}\right|^2=28\Rightarrow 4\left|\overrightarrow{u}\right|^2+\left|\overrightarrow{v}\right|^2+4\overrightarrow{u}\cdot\overrightarrow{v}=28$

$$\Rightarrow 36+16+4\left(3\cdot4\cdot\cos\theta\right)=28 \quad \Rightarrow \cos\theta=-\frac{1}{2},\ \therefore\theta=120^\circ\ .$$

4.設$\overrightarrow{a}$ 與$\overrightarrow{b}$ 是平面向量，$\left|\overrightarrow{a}\right|=3$，$\left|\overrightarrow{b}\right|=2$，且$\left|\overrightarrow{a}+2\overrightarrow{b}\right|=\sqrt{21}$，則$\left|3\overrightarrow{a}-\overrightarrow{b}\right|=$____________ .

解答 $\sqrt{91}$

解析 $\left|\overrightarrow{a}+2\overrightarrow{b}\right|^2=21\Rightarrow 9+4\cdot4+4\overrightarrow{a}\cdot\overrightarrow{b}=21$，$\therefore\overrightarrow{a}\cdot\overrightarrow{b}=-1$，

$$\left|3\overrightarrow{a}-\overrightarrow{b}\right|^2=9\cdot9+4-6(-1)=91,\ \therefore\left|3\overrightarrow{a}-\overrightarrow{b}\right|=\sqrt{91}\ .$$

5.△ABC 中，$\overline{AB}=2$，$\overline{BC}=4$，$\overline{AC}=3$，則(1)$\overrightarrow{AB}\cdot\overrightarrow{AC}=$__________；(2)$\left|\overrightarrow{AB}+2\overrightarrow{AC}\right|=$___________ .

解答 (1) $-\frac{3}{2}$;(2) $\sqrt{34}$

解析 (1) $2\cdot3\cdot\cos A=2\cdot3\cdot\frac{2^2+3^2-4^2}{2\cdot2\cdot3}=-\frac{3}{2}$.

$$(2)\sqrt{\left|\overrightarrow{AB}+2\overrightarrow{AC}\right|^2}=\sqrt{4+4\cdot9+4\left(\frac{-3}{2}\right)}=\sqrt{4+36-6}=\sqrt{34}\ .$$

6. $\overrightarrow{a}$ ，$\overrightarrow{b}$ 為平面上二向量，$\overrightarrow{a}\perp\overrightarrow{b}$ ，$\left|\overrightarrow{a}\right|=2$，$\left|\overrightarrow{b}\right|=1$，若$\overrightarrow{a}+\left(t^2+3\right)\overrightarrow{b}$ 與$\overrightarrow{a}+t\overrightarrow{b}$ 互相垂直，

則 $t=$____________ .

解答 -1

解析 $\because\left[\overrightarrow{a}+\left(t^2+3\right)\overrightarrow{b}\right]\perp\left[\overrightarrow{a}+t\,\overrightarrow{b}\right]$,

$\therefore\left[\overrightarrow{a}+\left(t^2+3\right)\overrightarrow{b}\right]\cdot\left[\overrightarrow{a}+t\,\overrightarrow{b}\right]=0$

$\Rightarrow\left|\overrightarrow{a}\right|^2+t\left(t^2+3\right)\left|\overrightarrow{b}\right|^2=0$（$\overrightarrow{a}\perp\overrightarrow{b}$，$\therefore\overrightarrow{a}\cdot\overrightarrow{b}=0$）$\Rightarrow 4+t\left(t^2+3\right)\cdot 1=0$

$\Rightarrow t^3+3t+4=0\Rightarrow\left(t+1\right)\left(t^2-t+4\right)=0$

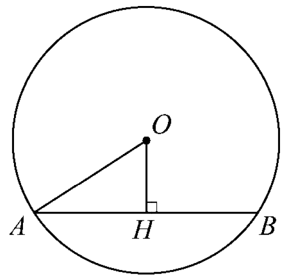
$\Rightarrow t$ 為實數，$\therefore t=-1$.

7.一圓之圓心為 O， $\overline{AB}$ 為一弦，若 $\overline{AB}=8$，則 $\overrightarrow{OA}\cdot\overrightarrow{AB}=$____________ .

解答 -32

解析

作 $\overline{OH}\perp\overline{AB}$，



$\overrightarrow{OA}\cdot\overrightarrow{AB}=-\overrightarrow{AO}\cdot\overrightarrow{AB}$

$=-\left|\overrightarrow{AO}\right|\cdot\left|\overrightarrow{AB}\right|\cdot\cos\angle OAB=-\left|\overrightarrow{AB}\right|\cdot\left|\overrightarrow{AO}\right|\cdot\cos\angle OAB$

$=-\left|\overrightarrow{AB}\right|\cdot\frac{1}{2}\left|\overrightarrow{AB}\right|=-8\cdot 4=-32$.

8.△ ABC 中，$\overline{AB}=3$，$\overline{AC}=4$，$\angle BAC=60^\circ$，求

(1) $\overline{BC}=$____________；(2) $\overrightarrow{AB}\cdot\overrightarrow{AC}=$____________；(3) $\overrightarrow{CA}\cdot\overrightarrow{CB}=$____________ .

解答 (1) $\sqrt{13}$;(2) 6 ;(3)10

解析 (1) $\overrightarrow{BC}=\overrightarrow{AC}-\overrightarrow{AB}$

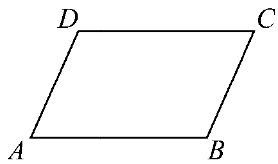
$\left|\overrightarrow{BC}\right|^2=\left|\overrightarrow{AC}-\overrightarrow{AB}\right|^2=\left|\overrightarrow{AC}\right|^2+\left|\overrightarrow{AB}\right|^2-2\left|\overrightarrow{AC}\right|\cdot\left|\overrightarrow{AB}\right|\cdot\cos 60^\circ\quad=16+9-2\cdot 4\cdot 3\cdot\frac{1}{2}=13$

$\therefore\left|\overrightarrow{BC}\right|=\sqrt{13}$，即 $\overline{BC}=\sqrt{13}$.

(2) $\overrightarrow{AB}\cdot\overrightarrow{AC}=\left|\overrightarrow{AB}\right|\cdot\left|\overrightarrow{AC}\right|\cdot\cos 60^\circ=3\cdot 4\cdot\frac{1}{2}=6$.

(3) $\overrightarrow{CA}\cdot\overrightarrow{CB}=\dfrac{\left|\overrightarrow{CA}\right|^2+\left|\overrightarrow{CB}\right|^2-\left|\overrightarrow{AB}\right|^2}{2}=\dfrac{16+13-9}{2}=10$.

9.設平行四邊形 $ABCD$ 中，已知 $\overline{AB}=8$，$\overrightarrow{AC}\cdot\overrightarrow{BD}=20$，則 $\overline{BC}$ 之長為_________ .



解答 $2\sqrt{21}$

解析

$$\overrightarrow{AC}\cdot\overrightarrow{BD}=\left(\overrightarrow{AB}+\overrightarrow{BC}\right)\cdot\left(\overrightarrow{AD}-\overrightarrow{AB}\right)=\left(\overrightarrow{AD}+\overrightarrow{AB}\right)\cdot\left(\overrightarrow{AD}-\overrightarrow{AB}\right)\quad=\left|\overrightarrow{AD}\right|^2-\left|\overrightarrow{AB}\right|^2,$$

$20=\left|\overrightarrow{AD}\right|^2-64\Rightarrow\left|\overrightarrow{AD}\right|^2=84$，$\left|\overrightarrow{AD}\right|=2\sqrt{21}$，即 $\overline{BC}=2\sqrt{21}$.

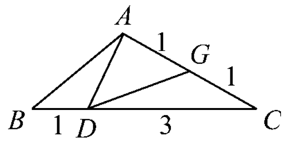
10.$\triangle ABC$ 中，D 為 $\overline{BC}$ 上一點且 $\overline{CD}=3\overline{BD}$，$G$ 為 $\overline{AC}$ 中點，若 $\overrightarrow{GD}=r\overrightarrow{AB}+s\overrightarrow{AC}$，$r$ 、s 為實數，則數對 $(r,s)=$____________ .

解答 $\left(\frac{3}{4},-\frac{1}{4}\right)$

解析

$$\overrightarrow{GD}=\overrightarrow{AD}-\overrightarrow{AG}=\left(\frac{3}{4}\overrightarrow{AB}+\frac{1}{4}\overrightarrow{AC}\right)-\frac{1}{2}\overrightarrow{AC}=\frac{3}{4}\overrightarrow{AB}-\frac{1}{4}\overrightarrow{AC},$$

故 $(r,s)=\left(\frac{3}{4},-\frac{1}{4}\right)$.



11.若 D 為 $\triangle ABC$ 內部一點，且 $\triangle ABD$ 的面積：$\triangle ACD$ 的面積 $=2:3$，又 $\overline{AD}$ 之延長線與 $\overline{BC}$ 相交於 E，若 $\overrightarrow{AE}=x\overrightarrow{AB}+y\overrightarrow{AC}$，則數對 $(x,y)=$____________ .

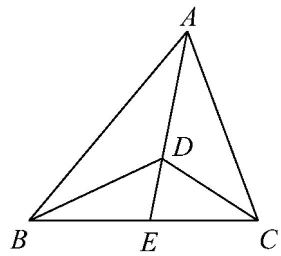
解答 $\left(\frac{3}{5},\frac{2}{5}\right)$

解析

$\overline{BE}:\overline{CE}=\triangle ABD:\triangle ACD=2:3$,

$\therefore\ \overrightarrow{AE}=\frac{3}{5}\overrightarrow{AB}+\frac{2}{5}\overrightarrow{AC}$,

故 $(x,y)=\left(\frac{3}{5},\frac{2}{5}\right)$.



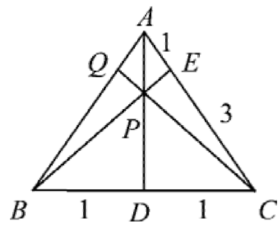
12.$\triangle ABC$ 中，D 為 $\overline{BC}$ 中點，E 在 $\overline{AC}$ 上，且 $\overline{AE}$ ：$\overline{EC}=1$ ：3，$\overline{AD}$ 交 $\overline{BE}$ 於點 P，$\overline{CP}$ 之延長線與 $\overline{AB}$ 交於 Q 點，則

(1)若 $\overrightarrow{CP}=x\overrightarrow{CA}+y\overrightarrow{CB}$，則數對 $(x,y)=$____________；(2) $\overline{CP}$ ：$\overline{PQ}=$____________ .

解答 (1) $\left(\frac{3}{5},\frac{1}{5}\right)$;(2)4：1

解析

(1) $\overrightarrow{CP}=x\overrightarrow{CA}+y\overrightarrow{CB}$，



∵ $\begin{cases}\overrightarrow{CP}=x\overrightarrow{CA}+y\left(2\overrightarrow{CD}\right)\\ \overrightarrow{CP}=x\left(\dfrac{4}{3}\overrightarrow{CE}\right)+y\overrightarrow{CB}\end{cases}$，

A、P、D共線且B、P、E共線，

$\therefore \begin{cases}x+2y=1\\ \dfrac{4}{3}x+y=1\end{cases}\Rightarrow (x,y)=\left(\dfrac{3}{5},\dfrac{1}{5}\right)$.

(2)設$\overrightarrow{CQ}=t\overrightarrow{CP}=\dfrac{3t}{5}\overrightarrow{CA}+\dfrac{t}{5}\overrightarrow{CB}$

∵ A、Q、B共線，∴ $\dfrac{3t}{5}+\dfrac{t}{5}=1\Rightarrow t=\dfrac{5}{4}$，

即$\overrightarrow{CQ}=\dfrac{5}{4}\overrightarrow{CP}$，故$\overline{CP}:\overline{PQ}=4:1$.

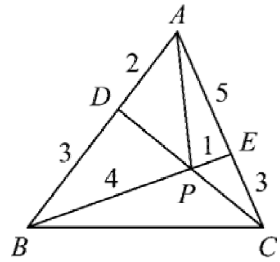
13.△ABC中，D在$\overline{AB}$上且$\overline{AD}:\overline{DB}=2:3$，$E$在$\overline{AC}$上，且$\overline{AE}:\overline{EC}=5:3$，又$\overline{BE}$與$\overline{CD}$交於$P$點，若$\overrightarrow{AP}=x\overrightarrow{AB}+y\overrightarrow{AC}$，其中$x$、$y$皆為實數，則數對$(x,y)=$____________.

解答 $\left(\dfrac{1}{5},\dfrac{1}{2}\right)$

解析 SOL 一

由孟氏定理：$\dfrac{3}{2}\cdot\dfrac{8}{3}\cdot\dfrac{\overline{EP}}{\overline{PB}}=1$，

$\therefore \overline{EP}:\overline{PB}=1:4$，$\therefore \overrightarrow{AP}=\dfrac{1}{5}\overrightarrow{AB}+\dfrac{4}{5}\overrightarrow{AE}=\dfrac{1}{5}\overrightarrow{AB}+\dfrac{4}{5}\left(\dfrac{5}{8}\overrightarrow{AC}\right)$，



故$(x,y)=\left(\dfrac{1}{5},\dfrac{1}{2}\right)$.

SOL 二

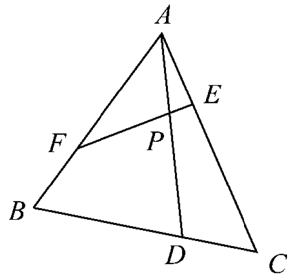
設$\overline{BP}:\overline{PE}=x:1$

△ABE中，$\overrightarrow{AP}=\dfrac{1}{x+1}\overrightarrow{AB}+\dfrac{x}{x+1}\overrightarrow{AE}=\dfrac{1}{x+1}\cdot\dfrac{5}{2}\overrightarrow{AP}+\dfrac{x}{x+1}\cdot\dfrac{5}{8}\overrightarrow{AC}$

∵ D、P、C三點共線，∴ $\dfrac{5}{2(x+1)}+\dfrac{5x}{8(x+1)}=1\Rightarrow x=4$

$\therefore \overrightarrow{AP}=\dfrac{1}{5}\overrightarrow{AB}+\dfrac{4}{5}\overrightarrow{AE}=\dfrac{1}{5}\overrightarrow{AB}+\dfrac{4}{5}\cdot\dfrac{5}{8}\overrightarrow{AC}=\dfrac{1}{5}\overrightarrow{AB}+\dfrac{1}{2}\overrightarrow{AC}$.

14.如圖，D、E、F 依次分別為 $\overline{BC}$、$\overline{CA}$、$\overline{AB}$ 上的點，$\overline{AF}:\overline{FB}=\overline{BD}:\overline{DC}=\overline{CE}:\overline{EA}=2:1$，而 $\overline{AD}$ 與 $\overline{EF}$ 交於 P，設 $\overrightarrow{AP}=x\overrightarrow{AB}+y\overrightarrow{AC}$，則數對 $(x,y)=$____________.



解答 $\left(\dfrac{2}{15},\dfrac{4}{15}\right)$

解析 $\overrightarrow{AD}=\dfrac{1}{3}\overrightarrow{AB}+\dfrac{2}{3}\overrightarrow{AC}$，

又

$\overrightarrow{AP}=t\overrightarrow{AD}=\dfrac{1}{3}t\overrightarrow{AB}+\dfrac{2}{3}t\overrightarrow{AC}=\dfrac{1}{3}t\left(\dfrac{3}{2}\overrightarrow{AF}\right)+\dfrac{2}{3}t\left(3\overrightarrow{AE}\right)=\left(\dfrac{1}{2}t\right)\overrightarrow{AF}+(2t)\overrightarrow{AE}$，

$\because F$、P、E 三點共線，$\therefore \dfrac{1}{2}t+2t=1\Rightarrow t=\dfrac{2}{5}$，

$\because \overrightarrow{AP}=\dfrac{2}{5}\overrightarrow{AD}=\dfrac{2}{5}\left(\dfrac{1}{3}\overrightarrow{AB}+\dfrac{2}{3}\overrightarrow{AC}\right)=\dfrac{2}{15}\overrightarrow{AB}+\dfrac{4}{15}\overrightarrow{AC}$，$\therefore (x,y)=\left(\dfrac{2}{15},\dfrac{4}{15}\right)$.

15.設 G 為△ABC 之重心，求

(1)若 $\overrightarrow{AG}=x\overrightarrow{AB}+y\overrightarrow{AC}$，則數對 $(x,y)=$____________;

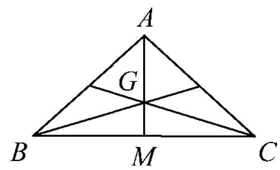
(2)若 $\overrightarrow{AG}=x\overrightarrow{AB}+y\overrightarrow{BC}$，則數對 $(x,y)=$____________;

(3)若 $\overrightarrow{AG}=x\overrightarrow{BC}+y\overrightarrow{CA}$，則數對 $(x,y)=$____________.

解答 (1)$\left(\dfrac{1}{3},\dfrac{1}{3}\right)$;(2)$\left(\dfrac{2}{3},\dfrac{1}{3}\right)$;(3)$\left(-\dfrac{1}{3},-\dfrac{2}{3}\right)$

解析

(1)△ABC 中，



$\overrightarrow{AM}=\dfrac{1}{2}\overrightarrow{AB}+\dfrac{1}{2}\overrightarrow{AC}$，

$\overrightarrow{AG}=\dfrac{2}{3}\overrightarrow{AM}=\dfrac{2}{3}\left(\dfrac{1}{2}\overrightarrow{AB}+\dfrac{1}{2}\overrightarrow{AC}\right)=\dfrac{1}{3}\overrightarrow{AB}+\dfrac{1}{3}\overrightarrow{AC}$，

$\therefore (x,y)=\left(\dfrac{1}{3},\dfrac{1}{3}\right)$.

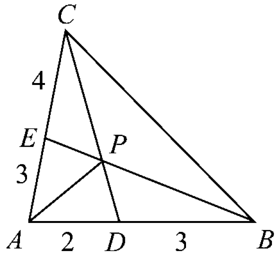
(2)由(1)，$\overrightarrow{AG}=\dfrac{1}{3}\overrightarrow{AB}+\dfrac{1}{3}\overrightarrow{AC}=\dfrac{1}{3}\overrightarrow{AB}+\dfrac{1}{3}\left(\overrightarrow{AB}+\overrightarrow{BC}\right)=\dfrac{2}{3}\overrightarrow{AB}+\dfrac{1}{3}\overrightarrow{BC}$，$\therefore (x,y)=\left(\dfrac{2}{3},\dfrac{1}{3}\right)$.

(3)由(2)，$\overrightarrow{AG}=\dfrac{2}{3}\overrightarrow{AB}+\dfrac{1}{3}\overrightarrow{BC}=\dfrac{2}{3}\left(\overrightarrow{AC}+\overrightarrow{CB}\right)+\dfrac{1}{3}\overrightarrow{BC}=-\dfrac{2}{3}\overrightarrow{CA}-\dfrac{2}{3}\overrightarrow{BC}+\dfrac{1}{3}\overrightarrow{BC}$

$=-\dfrac{1}{3}\overrightarrow{BC}-\dfrac{2}{3}\overrightarrow{CA}$，　$\therefore (x,y)=\left(-\dfrac{1}{3},-\dfrac{2}{3}\right)$.

16.

如右圖，$\overline{AD}:\overline{DB}=2:3$，$\overline{AE}:\overline{EC}=3:4$，$\overline{BE}$與$\overline{CD}$交於 P，若 $\overrightarrow{AP}=x\overrightarrow{AB}+y\overrightarrow{AC}$，則 $(x,y)=$＿＿＿＿＿＿＿＿．



解答　$\left(\dfrac{8}{29},\dfrac{9}{29}\right)$

解析　$\overrightarrow{AP}=x\overrightarrow{AB}+y\overrightarrow{AC}\Rightarrow\begin{cases}\overrightarrow{AP}=x\overrightarrow{AB}+y\left(\dfrac{7}{3}\overrightarrow{AE}\right)\\\overrightarrow{AP}=x\left(\dfrac{5}{2}\overrightarrow{AD}\right)+y\overrightarrow{AC}\end{cases}$，

$\therefore B$、P、E 共線且 C、P、D 共線，$\therefore\begin{cases}x+\dfrac{7}{3}y=1\\\dfrac{5}{2}x+y=1\end{cases}\Rightarrow(x,y)=\left(\dfrac{8}{29},\dfrac{9}{29}\right)$．

17.直線上三點 A、B、C，$A-B-C$，若 $3\overline{AB}=2\overline{BC}$，$O$ 為任一點，求

(1)若 $\overrightarrow{OA}=x\overrightarrow{OB}+y\overrightarrow{OC}$，則數對 $(x,y)=$＿＿＿＿＿＿＿＿；

(2)若 $\overrightarrow{OB}=h\overrightarrow{OA}+k\overrightarrow{OC}$，則數對 $(h,k)=$＿＿＿＿＿＿＿．

解答　(1)$\left(\dfrac{5}{3},-\dfrac{2}{3}\right)$;(2)$\left(\dfrac{3}{5},\dfrac{2}{5}\right)$

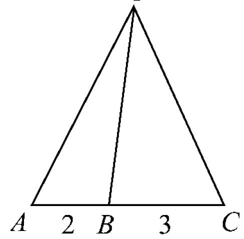
解析　$\because 3\overline{AB}=2\overline{BC}\Rightarrow\dfrac{\overline{AB}}{\overline{BC}}=\dfrac{2}{3}$，

由分點公式 $\overrightarrow{OB}=\dfrac{3}{5}\overrightarrow{OA}+\dfrac{2}{5}\overrightarrow{OC}$，

$\dfrac{3}{5}\overrightarrow{OA}=\overrightarrow{OB}-\dfrac{2}{5}\overrightarrow{OC}\Rightarrow\overrightarrow{OA}=\dfrac{5}{3}\overrightarrow{OB}-\dfrac{2}{3}\overrightarrow{OC}$，

$\therefore$(1) $(x,y)=\left(\dfrac{5}{3},-\dfrac{2}{3}\right)$．

(2) $(h,k)=\left(\dfrac{3}{5},\dfrac{2}{5}\right)$．



18. A、B、C 三點不共線，x、y 為實數，若 $(x+1)\overrightarrow{AB}+(3y-6)\overrightarrow{AC}+(2x-y)\overrightarrow{BC}=\overrightarrow{0}$，則數對 $(x,y)=$

______ .

解答 $(2,1)$

解析 $(x+1)\overrightarrow{AB}+(3y-6)\overrightarrow{AC}+(2x-y)\left(\overrightarrow{AC}-\overrightarrow{AB}\right)=\overrightarrow{0}$,

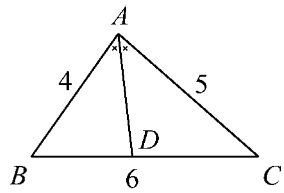
$(x+1-2x+y)\overrightarrow{AB}+(3y-6+2x-y)\overrightarrow{AC}=\overrightarrow{0}$,

$\therefore \begin{cases}-x+y=-1\\2x+2y=6\end{cases}\Rightarrow \begin{matrix}x=2\\y=1\end{matrix}$, $\therefore (x,y)=(2,1)$.

19.若$\triangle ABC$中,$\overline{AB}=4$,$\overline{AC}=5$,$\overline{BC}=6$且$\angle A$的角平分線$\overline{AD}$交$\overline{BC}$於D點,則$\left|\overrightarrow{AD}\right|=$__________.

解答 $\dfrac{10}{3}$

解析



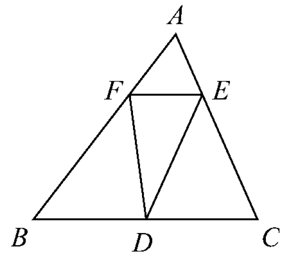
$\because \dfrac{\overline{BD}}{\overline{DC}}=\dfrac{\overline{AB}}{\overline{AC}}=\dfrac{4}{5}$,

$\therefore \overrightarrow{AD}=\dfrac{5}{9}\overrightarrow{AB}+\dfrac{4}{9}\overrightarrow{AC}$,

$\left|\overrightarrow{AD}\right|^2=\left|\dfrac{5}{9}\overrightarrow{AB}+\dfrac{4}{9}\overrightarrow{AC}\right|^2=\dfrac{25}{81}\left|\overrightarrow{AB}\right|^2+\dfrac{16}{81}\left|\overrightarrow{AC}\right|^2+\dfrac{40}{81}\overrightarrow{AB}\cdot\overrightarrow{AC}$

$=\dfrac{25}{81}\cdot 16+\dfrac{16}{81}\cdot 25+\dfrac{40}{81}\left(4\cdot 5\cdot\dfrac{16+25-36}{2\cdot 4\cdot 5}\right)=\dfrac{900}{81}=\dfrac{100}{9}$, $\therefore \left|\overrightarrow{AD}\right|=\dfrac{10}{3}$.

20.



在$\triangle ABC$之三邊上分別取D、E、F三點,使$\overrightarrow{AF}=\dfrac{1}{2}\overrightarrow{FB}$, $\overrightarrow{AE}=\dfrac{1}{3}\overrightarrow{AC}$,

且D為$\overline{BC}$之中點,若G為$\triangle DEF$之重心且$\overrightarrow{AG}=x\overrightarrow{AB}+y\overrightarrow{AC}$, 則

(1) $x=$______________;(2) $y=$______________ .

解答 (1) $\dfrac{5}{18}$;(2) $\dfrac{5}{18}$

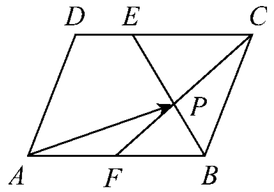
解析 $\because G$為$\triangle DEF$之重心,

$\therefore \overrightarrow{AG}=\dfrac{1}{3}\overrightarrow{AD}+\dfrac{1}{3}\overrightarrow{AE}+\dfrac{1}{3}\overrightarrow{AF}=\dfrac{1}{3}\left(\dfrac{1}{2}\overrightarrow{AB}+\dfrac{1}{2}\overrightarrow{AC}\right)+\dfrac{1}{3}\cdot\dfrac{1}{3}\overrightarrow{AC}+\dfrac{1}{3}\cdot\dfrac{1}{3}\overrightarrow{AB}=\dfrac{5}{18}\overrightarrow{AB}+\dfrac{5}{18}\overrightarrow{AC}$,

$\therefore x=\dfrac{5}{18}$, $y=\dfrac{5}{18}$.

21.

如圖，平行四邊形 $ABCD$ 中，$\overline{DE}=\frac{1}{3}\overline{DC}$，$F$ 為 $\overline{AB}$ 中點，

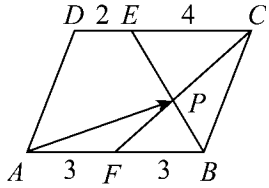


$\overline{BE}$、$\overline{CF}$ 交於 P，若 $\overrightarrow{AP}=x\overrightarrow{AB}+y\overrightarrow{AD}$，則數對

$(x, y)=$ ____________ .

解答 $\left(\frac{5}{7},\frac{3}{7}\right)$

解析 由圖$\Rightarrow \overline{BP}:\overline{PE}=3:4$,



$\therefore \overrightarrow{AP}=\frac{4}{7}\overrightarrow{AB}+\frac{3}{7}\overrightarrow{AE}=\frac{4}{7}\overrightarrow{AB}+\frac{3}{7}\left(\overrightarrow{AD}+\overrightarrow{DE}\right)$

$=\frac{4}{7}\overrightarrow{AB}+\frac{3}{7}\overrightarrow{AD}+\frac{3}{7}\left(\frac{1}{3}\overrightarrow{AB}\right)=\frac{5}{7}\overrightarrow{AB}+\frac{3}{7}\overrightarrow{AD}$，

$\therefore (x, y)=\left(\frac{5}{7},\frac{3}{7}\right)$.

22. I 為$\triangle ABC$ 的內心，$a=3$，$b=5$，$c=6$，若 $\overrightarrow{BI}=x\overrightarrow{BA}+y\overrightarrow{BC}$，則數對 $(x, y)=$ ____________ .

解答 $\left(\frac{3}{14},\frac{3}{7}\right)$

解析 $\overrightarrow{BI}=\frac{a}{a+b+c}\overrightarrow{BA}+\frac{c}{a+b+c}\overrightarrow{BC}=\frac{3}{14}\overrightarrow{BA}+\frac{6}{14}\overrightarrow{BC}$，

$\therefore (x, y)=\left(\frac{3}{14},\frac{3}{7}\right)$.