

高雄市明誠中學 高一數學平時測驗 日期：99.10.20				
範圍	2-2 餘式、因式定理	班級	一年____班	姓名
		座號		

一、填充題 (每題'10分)

1 設 $x^3 + ax^2 - 5x - 2$ 可被 $x^2 + 3x + b$ 整除, 則 $(a, b) = \underline{\hspace{2cm}}$.

解答 (1, 1) 或 $(\frac{10}{3}, -6)$

解析

$$\begin{array}{r}
 1 + (a-3) \\
 1+3+b \overline{) 1 + \quad a - \quad 5 - \quad 2} \\
 \underline{1 + \quad 3 + \quad b} \\
 (a-3) + (-5-b) - \quad 2 \\
 \underline{(a-3) + 3(a-3) + b(a-3)} \\
 0
 \end{array}$$

$$\Rightarrow \begin{cases} -5-b=3a-9 \Rightarrow b=4-3a \text{ 代入下式} \\ -2=b(a-3) \end{cases}$$

$$\Rightarrow 3a^2 - 13a + 10 = 0 \Rightarrow a=1 \text{ 或 } \frac{10}{3} \Rightarrow b=1 \text{ 或 } -6, \therefore (a, b) = (1, 1) \text{ 或 } (\frac{10}{3}, -6)$$

2. 設 $f(x)$ 為一多項式, 若 $(x+1)f(x)$ 除以 $x^2 + x + 1$ 的餘式為 $2x-1$, 則 $f(x)$ 除以 $x^2 + x + 1$ 的餘式為 $\underline{\hspace{2cm}}$.

解答 $3x+2$

解析 設 $f(x)$ 被 $x^2 + x + 1$ 除之餘式為 $ax + b$, 由除法原理知

$$f(x) = (x^2 + x + 1)Q(x) + (ax + b)$$

$$\begin{aligned}
 \text{同乘 } x^2 + x + 1 \Rightarrow (x+1)f(x) &= (x^2 + x + 1)Q(x)(x+1) + (ax+b)(x+1) \\
 &= (x^2 + x + 1)Q(x)(x+1) + a(x^2 + x + 1) + bx + (b-a) \\
 &= (x^2 + x + 1)[Q(x)(x+1) + a] + bx + (b-a),
 \end{aligned}$$

表 $(x+1)f(x)$ 除以 $x^2 + x + 1$ 的餘式為 $bx + (b-a)$,

得 $bx + (b-a) = 2x - 1$, 故 $b=2$, $a=3$, 所求之餘式為 $ax + b = 3x + 2$.

3. 將一多項式 $f(x) = x^3 - 3x^2 + x + 3$ 表示成 $a(x-2)^3 + b(x-2)^2 + c(x-2) + d$ 的形式, 其中 a, b, c, d 皆為實數 .

(1) 求 $(a, b, c, d) = \underline{\hspace{2cm}}$.

(2) 利用(1)之結果計算 $f(1.99)$ 之近似值至小數第四位為 $\underline{\hspace{2cm}}$.

解答 (1) (1, 3, 1, 1); (2) 0.9903

解析 (1) $f(x) = x^3 - 3x^2 + x + 3 = a(x-2)^3 + b(x-2)^2 + c(x-2) + d$

$$\begin{array}{r}
 1-3+1+3 \quad | \quad 2 \\
 +2-2-2 \\
 \hline
 1-1-1 \oplus 1 \longrightarrow d \\
 +2+2 \\
 \hline
 1+1 \oplus 1 \longrightarrow c \\
 +2 \\
 \hline
 1 \oplus 3 \longrightarrow b \\
 \uparrow \\
 a
 \end{array}$$

$$\Rightarrow (a, b, c, d) = (1, 3, 1, 1), \text{ 即 } f(x) = 1 + (x-2) + 3(x-2)^2 + (x-2)^3$$

$$(2) f(1.99) = 1 + (1.99-2) + 3(1.99-2)^2 + (1.99-2)^3 \doteq 1 - 0.01 + 3 \times 0.0001 \doteq 0.9903$$

4. 求 $81(0.666)^4 - 54(0.666)^3 - 63(0.666)^2 + 39(0.666) + 5$ 之近似值到小數點後第三位_____ (第四位以後四捨五入) .

解答 3.014

解析 令 $f(x) = 81x^4 - 54x^3 - 63x^2 + 39x + 5$

$$\text{由綜合除法可知 } f(x) = 1 \cdot (3x-2)^4 + 6(3x-2)^3 + 5(3x-2)^2 - 7(3x-2) + 3$$

$$\because 3x-2 = 3(0.666) - 2 = 1.998 - 2 = -0.002$$

$$\text{故 } f(0.666) \doteq (-7) \times (-0.002) + 3 = 3.014 \text{ (只取後二項之值即可)}$$

5. 設 x 的多項式 $3(x-1)^3 + 4(x-1)^2 + 2 = a(x-1)(x-2)(x+1) + b(x-1)(x-2) + c(x-2) + d$, 求 $(a, b, c, d) =$ _____ .

解答 (3, 1, 7, 9)

解析 $x=2$ 代入 $\Rightarrow 3+4+2=d \quad \therefore d=9$

$$x=1 \text{ 代入 } \Rightarrow 2=c \times (-1) + d \Rightarrow c=7$$

$$x=-1 \text{ 代入 } \Rightarrow 3 \times (-2)^3 + 4 \times (-2)^2 + 2 = b(-2)(-3) + c(-3) + d$$

$$\text{即 } 6b - 3c + d = -6 \Rightarrow b=1$$

$$x=0 \text{ 代入 } \Rightarrow 3 \times (-1)^3 + 4 \times (-1)^2 + 2 = a(-1)(-2) \times 1 + b(-1)(-2) + c(-2) + d$$

$$\text{即 } 2a + 2b - 2c + d = 3 \Rightarrow 2a + 2 - 14 + 9 = 3 \Rightarrow a=3$$

6. 設 $\deg f(x) = 3, f(123) = 5, f(124) = 6, f(125) = 25, f(126) = 44$, 則 $f(122)$ 之值為_____ .

解答 40

解析 【解 1】

$$\text{設 } f(x) = a(x-125)^3 + b(x-125)^2 + c(x-125) + 25$$

$$\because \begin{cases} f(123) = 5 \\ f(124) = 6 \\ f(126) = 44 \end{cases} \Rightarrow \begin{cases} -8a + 4b - 2c + 25 = 5 \\ -a + b - c + 25 = 6 \\ a + b + c + 25 = 44 \end{cases} \Rightarrow \begin{cases} 4a - 2b + c = 10 \\ a - b + c = 19 \\ a + b + c = 19 \end{cases}$$

$$\therefore b=0, a=-3, c=22 \quad \therefore f(x) = -3(x-125)^3 + 22(x-125) + 25$$

$$\therefore f(122) = (-3)(-27) + 22(-3) + 25 = 40$$

【解 2】

$$f(x) = 5 \times \frac{(x-124)(x-125)(x-126)}{(123-124)(123-125)(123-126)} + 6 \times \frac{(x-123)(x-125)(x-126)}{(124-123)(124-125)(124-126)}$$

$$\begin{aligned}
& +25 \times \frac{(x-123)(x-124)(x-126)}{(125-123)(125-124)(125-126)} + 44 \times \frac{(x-123)(x-124)(x-125)}{(126-123)(126-124)(126-125)} \\
\Rightarrow f(122) &= 5 \times \frac{(-2)(-3)(-4)}{(-1)(-2)(-3)} + 6 \times \frac{(-1)(-3)(-4)}{(1)(-1)(-2)} + 25 \times \frac{(-1)(-2)(-4)}{(2)(1)(-1)} + 44 \times \frac{(-1)(-2)(-3)}{(3)(2)(1)} \\
&= 20 - 36 + 100 - 44 = 40
\end{aligned}$$

7. 設 $f(x) = 351x^5 - 692x^4 - 23x^3 + 9x^2 - 36x + 50$, 則 $f(2) =$ _____ .

解答 -10

解析

$$\begin{array}{r}
351 - 692 - 23 + 9 - 36 + 50 \quad | \underline{2} \\
+ 702 + 20 - 6 + 6 - 60 \\
\hline
351 + 10 - 3 + 3 - 30 - 10
\end{array}$$

由上綜合除法可知：餘式 $r = f(2) = -10$

8. 設 $f(x)$ 與 $g(x)$ 為實係數多項式, 以 $x^2 - 3x + 2$ 除 $f(x)$ 得餘式為 $3x - 4$, 以 $x - 1$ 除 $g(x)$ 得餘式為 5 , 則以 $x - 1$ 除 $f(x) + g(x)$ 的餘式為 _____ .

解答 4

解析

$$f(x) = (x^2 - 3x + 2)q(x) + 3x - 4 \Rightarrow f(1) = 0 \cdot q(1) + 3 - 4 = -1$$

$$\text{以 } x - 1 \text{ 除 } g(x) \text{ 之餘式為 } 5 \Rightarrow g(1) = 5$$

$$\text{以 } x - 1 \text{ 除 } f(x) + g(x) \text{ 之餘式為 } f(1) + g(1) = (-1) + 5 = 4$$

9. 若三次多項式 $g(x)$ 的 $g(-1) = g(0) = g(2) = 0$, $g(1) = 4$, 試問

(1) $g(x) =$ _____ . (2) 若多項式 $h(x) = x^4 - x^2 + 1$, 則 $3g(x) - 4h(x)$ 被 $x - 1$ 除的餘式為

解答 (1) $-2x(x+1)(x-2)$; (2) 8

解析

(1) 由 $g(-1) = g(0) = g(2) = 0$, $\deg g(x) = 3$, 可設 $g(x) = ax(x+1)(x-2)$

$$\text{又 } g(1) = a \times 2 \times (-1) = 4 \Rightarrow a = -2, \text{ 故 } g(x) = -2x(x+1)(x-2)$$

$$(2) \text{ 令 } F(x) = 3g(x) - 4h(x)$$

$$\text{則所求餘式為 } F(1) = 3g(1) - 4h(1) = 3 \times 4 - 4 \times (1 - 1 + 1) = 12 - 4 = 8$$

10. a, b 為常數, 若 $2x - 3$ 與 $3x + 1$ 均為 $ax^3 + bx^2 - 47x - 15$ 的因式, 則數對 $(a, b) =$ _____ .

解答 (24, 2)

解析

$$\text{令 } f(x) = ax^3 + bx^2 - 47x - 15$$

$$2x - 3 \mid f(x) \Rightarrow f\left(\frac{3}{2}\right) = 0 \Rightarrow \frac{27}{8}a + \frac{9}{4}b - \frac{141}{2} - 15 = 0 \Rightarrow 3a + 2b = 76 \cdots \cdots \textcircled{1}$$

$$3x + 1 \mid f(x) \Rightarrow f\left(-\frac{1}{3}\right) = 0 \Rightarrow -\frac{1}{27}a + \frac{1}{9}b + \frac{47}{3} - 15 = 0 \Rightarrow -a + 3b = -18 \cdots \cdots \textcircled{2}$$

$$\textcircled{1} + \textcircled{2} \times 3 \quad 11b = 76 - 54 = 22 \quad \therefore b = 2 \text{ 代入 } \textcircled{2} \text{ 得 } a = 24$$

11. 設 $f(x)$ 為實係數多項式, 以 $x - 1$ 除之, 餘式為 9 ; 以 $x - 2$ 除之, 餘式為 16 , 求

$f(x)$ 除以 $(x - 1)(x - 2)$ 的餘式為 _____ .

解答 $7x + 2$

解析 已知 $f(1) = 9, f(2) = 16$, 設 $f(x) = (x-1)(x-2)Q(x) + (ax+b)$

$$\begin{cases} f(1) = a + b = 9 \\ f(2) = 2a + b = 16 \end{cases} \Rightarrow \begin{cases} a = 7 \\ b = 2 \end{cases} \therefore \text{餘式} = 7x + 2$$

12. 若多項式 $f(x)$ 除以 $x^2 + 2x - 3$ 得餘式 $2x + 5$; 除以 $x^2 - 3x - 10$ 得餘式 $5x - 2$, 則 $f(x)$ 除以 $x^2 - 6x + 5$ 的餘式為_____.

解答 $4x + 3$

解析 已知 $\begin{cases} f(x) = (x^2 + 2x - 3)q_1(x) + 2x + 5 \\ f(x) = (x^2 - 3x - 10)q_2(x) + 5x - 2 \end{cases}$

$$\Rightarrow \begin{cases} f(x) = (x+3)(x-1)q_1(x) + 2x + 5 \\ f(x) = (x-5)(x+2)q_2(x) + 5x - 2 \end{cases} \Rightarrow \begin{cases} f(1) = 7 \\ f(5) = 23 \end{cases}$$

設 $f(x) = (x^2 - 6x + 5)Q(x) + R(x) = (x-5)(x-1)Q(x) + ax + b$

令 $x = 1 \Rightarrow f(1) = a + b = 7 \cdots \cdots \textcircled{1}$; $x = 5 \Rightarrow f(5) = 5a + b = 23 \cdots \cdots \textcircled{2}$

由 $\textcircled{1}, \textcircled{2}$ 得 $a = 4, b = 3$, 故餘式 $R(x) = ax + b = 4x + 3$

13. 多項式 $f(x)$, $f(x) \div (x-3)$ 之餘式為 2, $f(x) \div (2x^2 + 5x - 3)$ 之餘式為 $4x - 1$, 則

(1) $f(x) \div (2x^2 - 7x + 3)$ 之餘式為_____.

(2) $f(x) \div (x-3)(2x^2 + 5x - 3)$ 之餘式為_____.

解答 (1) $\frac{2}{5}x + \frac{4}{5}$; (2) $-\frac{3}{5}x^2 + \frac{5}{2}x - \frac{1}{10}$

解析 (1) $f(x) = (2x^2 + 5x - 3)q_1(x) + 4x - 1 = (2x-1)(x+3)q_1(x) + 4x - 1, f(\frac{1}{2}) = 4 \times \frac{1}{2} - 1 = 1$

又 $2x^2 - 7x + 3 = (2x-1)(x-3) \therefore$ 設 $f(x) = (2x-1)(x-3)q_2(x) + a(x-3) + 2$

$$f(\frac{1}{2}) = a(\frac{1}{2} - 3) + 2 = 1 \Rightarrow a = \frac{2}{5}$$

$$\therefore f(x) \div (2x^2 - 7x + 3) \text{ 的餘式為 } \frac{2}{5}(x-3) + 2 = \frac{2}{5}x + \frac{4}{5}$$

(2) 設 $f(x) = (x-3)(2x^2 + 5x - 3)q_3(x) + b(2x^2 + 5x - 3) + 4x - 1$

$$\therefore f(3) = 2 \Rightarrow b(2 \times 3^2 + 5 \times 3 - 3) + 4 \times 3 - 1 = 2 \Rightarrow 30b + 11 = 2, b = -\frac{3}{10}$$

$$\text{故 } f(x) \div (x-3)(2x^2 + 5x - 3) \text{ 的餘式為 } -\frac{3}{10}(2x^2 + 5x - 3) + 4x - 1 = -\frac{3}{5}x^2 + \frac{5}{2}x - \frac{1}{10}$$

14. 設 $\deg f(x) = 3$, 已知 $f(1) = f(2) = f(3) = 4, f(4) = 34$, 則 $f(x) =$ _____.

解答 $5(x-1)(x-2)(x-3) + 4$

解析 $\therefore f(1) = f(2) = f(3) = 4 \therefore f(x)$ 除以 $x-1, x-2, x-3$ 都餘 4, 又 $\deg f(x) = 3$

設 $f(x) = a(x-1)(x-2)(x-3) + 4$

$$\therefore f(4) = 34 \Rightarrow a(3)(2)(1) + 4 = 34, \therefore a = 5$$

$$\therefore f(x) = 5(x-1)(x-2)(x-3) + 4$$

15. 設 $\deg f(x) = 3$, 若 $f(2) = f(-1) = f(4) = 3$, $f(1) = -9$, 則 $f(0) =$ _____.

解答 -13

解析 $\deg f(x) = 3$, $f(2) = f(-1) = f(4) = 3 \Rightarrow f(x) = a(x-2)(x+1)(x-4) + 3$

$$\Rightarrow f(1) = a(-1)(2)(-3) + 3 = -9 \Rightarrow a = -2 \Rightarrow f(x) = -2(x-2)(x+1)(x-4) + 3$$

$$\therefore f(0) = -2(-2)(1)(-4) + 3 = -13$$

16. 設 $x^2 - 4x + 3$ 除多項式 $f(x)$, $g(x)$ 的餘式各為 $5x - 2$, $x - 3$; $x^2 + 3x + 2$ 除 $f(x)$, $g(x)$ 的餘式各為 $4x + 3$, $2x + 1$, 則 $x^2 - x - 6$ 除 $f(x) \cdot g(x)$ 的餘式為_____.

解答 $-3x + 9$

解析 $\because (x-1)(x-3)$ 除 $f(x)$, $g(x)$ 餘式各為 $5x-2$, $x-3 \Rightarrow \begin{cases} f(x) = (x-1)(x-3)Q_1(x) + 5x-2 \\ g(x) = (x-1)(x-3)Q_2(x) + x-3 \end{cases}$

$$\Rightarrow x-3 \text{ 除 } f(x), g(x) \text{ 餘式各為 } f(3) = 5 \times 3 - 2 = 13, g(3) = 3 - 3 = 0$$

$$\text{又 } (x+1)(x+2) \text{ 除 } f(x), g(x) \text{ 餘式各為 } 4x+3, 2x+1$$

$$\Rightarrow \begin{cases} f(x) = (x+1)(x+2)Q_1(x) + 4x+3 \\ g(x) = (x+1)(x+2)Q_2(x) + 2x+1 \end{cases}$$

$$\therefore x+2 \text{ 除 } f(x), g(x) \text{ 餘式各為 } f(-2) = 4 \times (-2) + 3 = -5, g(-2) = 2 \times (-2) + 1 = -3$$

$$\therefore x-3 \text{ 除 } f(x) \cdot g(x) \text{ 餘式為 } f(3)g(3) = 13 \cdot 0 = 0$$

$$x+2 \text{ 除 } f(x) \cdot g(x) \text{ 餘式為 } f(-2)g(-2) = (-5)(-3) = 15$$

$$\text{設 } f(x) \cdot g(x) = Q(x)(x+2)(x-3) + (ax+b)$$

$$\therefore x-3 \text{ 除 } f(x)g(x) \text{ 餘式} = 3a+b=0, \text{ 又 } x+2 \text{ 除 } f(x)g(x) \text{ 餘式} = -2a+b=15$$

$$\therefore a = -3, b = 9, \text{ 即所求餘式為 } -3x + 9$$

17. 多項式 $f(x) = x^{2000} + 3x^{90} - 5x^{18} + 7$ 除以 $x^3 - 1$ 之餘式為_____.

解答 $x^2 + 5$

解析 考慮 $f(x) = Q(x)(x^3 - 1) + r(x)$

$$\text{令 } x^3 - 1 = 0, \text{ 即令 } x^3 = 1, \text{ 可由 } f(x) \text{ 求得餘式 } r(x)$$

$$\therefore f(x) = (x^3)^{666}x^2 + 3(x^3)^{30} - 5(x^3)^6 + 7$$

$$\therefore f(x) \text{ 除以 } x^3 - 1 \text{ 之餘式為 } 1^{666}x^2 + 3(1)^{30} - 5(1)^6 + 7 = x^2 + 5$$

18. 以 $x-1$ 除多項式 $f(x)$ 餘 1, 以 $x^2 + x + 1$ 除 $f(x)$ 餘式 $-x-1$, 求以 $x^3 - 1$ 除 $f(x)$ 之餘式為_____.

解答 x^2

解析 $\because x^3 - 1 = (x-1)(x^2 + x + 1)$

$$f(x) = (x-1)Q_1(x) + 1, \text{ 即 } f(1) = 1; \text{ 又 } f(x) = (x^2 + x + 1)Q_2(x) - x - 1$$

$$\text{設 } f(x) = (x-1)(x^2 + x + 1)Q'(x) + a(x^2 + x + 1) - x - 1$$

$$\therefore f(1) = a \times (1+1+1) - 1 - 1 = 1 \Rightarrow a = 1 \therefore \text{餘式為 } (x^2 + x + 1) - x - 1 = x^2$$

19.若 $f(x) = x^2$, $g(x) = (a-1)x^2 + (b+2)x + (c-3)$, 且 $f(5) = g(5)$, $f(\sqrt{2}) = g(\sqrt{2})$, $f(100) = g(100)$,

求 (1) $a =$ _____ . (2) $b =$ _____ . (3) $c =$ _____ .

解答 (1)2;(2)-2;(3)3

解析 5, $\sqrt{2}$, 100 為方程式 $f(x) = g(x)$ 的三根

但 $f(x)$, $g(x)$ 至多為二次方程式 $\Rightarrow f(x) = g(x)$ (恆等式)

即 $x^2 = (a-1)x^2 + (b+2)x + (c-3) = x^2 + 0x + 0$

$\therefore a-1=1$ 且 $b+2=0$ 且 $c-3=0 \Rightarrow a=2$ 且 $b=-2$ 且 $c=3$

20.設 $f(x) = x^{17} + 4x^3 - 3x + 1$, 則:(1)以 $x^2 - x + 1$ 除 $f(x)$ 之餘式為 _____ .

(2)以 $x^4 - x^3 + x - 1$ 除 $f(x)$ 之餘式為 _____ .

解答 (1) $-4x - 2$; (2) $5x^3 - x^2 - 3x + 2$

解析 (1) 令 $x^2 - x + 1 = 0 \Rightarrow (x+1)(x^2 - x + 1) = x^3 + 1 = 0 \quad \therefore x^3 = -1$

餘式 $r(x) = (-1)^5 \times x^2 + 4 \times (-1) - 3x + 1$

$= -x^2 - 3x - 3 = -(x-1) - 3x - 3 = -4x - 2$

(2) $\therefore x^4 - x^3 + x - 1 = (x-1)(x^3 + 1)$ 且由(1)知: $x^3 + 1$ 除 $f(x)$ 的餘式為 $-x^2 - 3x - 3$

\therefore 可設 $f(x) = x^{17} + 4x^3 - 3x + 1 \cdots \cdots \textcircled{1}$

$= (x^3 + 1)Q_1(x) + (-x^2 - 3x - 3)$

$= (x-1)(x^3 + 1)Q_2(x) + k(x^3 + 1) + (-x^2 - 3x - 3) \cdots \cdots \textcircled{2}$

令 $x=1$ 代入 $\textcircled{1}\textcircled{2} \quad \therefore f(1) = 3 = k \times 2 + (-7) \quad \therefore k = 5$

$\Rightarrow r(x) = 5(x^3 + 1) + (-x^2 - 3x - 3) = 5x^3 - x^2 - 3x + 2$

21. 計算 $2\left(\frac{3+\sqrt{17}}{4}\right)^4 + \left(\frac{3+\sqrt{17}}{4}\right)^3 - \left(\frac{3+\sqrt{17}}{4}\right)^2 - 10\left(\frac{3+\sqrt{17}}{4}\right) + 2$ 之值為 _____ .

解答 $\frac{23+\sqrt{17}}{4}$

解析 令 $x = \frac{3+\sqrt{17}}{4} \Rightarrow 4x - 3 = \sqrt{17} \Rightarrow 16x^2 - 24x - 8 = 0 \quad \therefore 2x^2 - 3x - 1 = 0$

設 $f(x) = 2x^4 + x^3 - x^2 - 10x + 2 \Rightarrow$ 求值式 $= f\left(\frac{3+\sqrt{17}}{4}\right)$

$\therefore f(x) = (2x^2 - 3x - 1)(x^2 + 2x + 3) + (x + 5)$

$\therefore f\left(\frac{3+\sqrt{17}}{4}\right) = \frac{3+\sqrt{17}}{4} + 5 = \frac{23+\sqrt{17}}{4}$

$1+2+3$

$2-3-1 \overline{) 2+1-1-10+2}$

$2-3-1$

$4+0-10$

$4-6-2$

$6-8+2$

$6-9-3$

$1+5$

22. 設多項式 $(x+1)^6$ 除以 x^2+1 的餘式為 $ax+b$ ，則(1) $a =$ _____，(2) $b =$ _____。

解答 (1) -8 ; (2) 0

解析 設 $A = x^2 + 1$ ，則 $(x+1)^2 = x^2 + 2x + 1 = A + 2x$

$$\begin{aligned}(x+1)^6 &= [(x+1)^2]^3 = (A+2x)^3 = A^3 + 6xA^2 + 12x^2A + 8x^3 \\ &= A(A^2 + 6xA + 12x^2) + 8x(x^2 + 1) - 8x \\ &= A(A^2 + 6xA + 12x^2 + 8x) - 8x\end{aligned}$$

故 $a = -8$ ， $b = 0$

23. 設三次以上的多項式 $f(x)$ 可被 $(x+1)^2$ 整除，且 $f(x)$ 除以 $x-1$ 餘式為 8 ，則 $f(x)$ 除以 $(x+1)^2(x-1)$ 的餘式為_____。

解答 $2x^2 + 4x + 2$

解析 設 $f(x) = (x+1)^2(x-1)P(x) + a(x+1)^2$ ，

$$\text{因 } f(1) = 8 \Rightarrow f(1) = a(1+1)^2 = 8, \therefore a = 2,$$

故所求餘式為 $2(x+1)^2 = 2x^2 + 4x + 2$ 。