

計算題 (共 100 分)

1. 設  $0 < \theta < \frac{\pi}{2}$ ，試化簡  $\sqrt{1 - \sin\theta} - \sqrt{1 + \sin\theta}$ 。(25 分)

解：  $\sqrt{1 - \sin\theta} - \sqrt{1 + \sin\theta}$

$$= \sqrt{1 - \sin\left(2 \times \frac{\theta}{2}\right)} - \sqrt{1 + \sin\left(2 \times \frac{\theta}{2}\right)}$$

$$= \sqrt{\sin^2 \frac{\theta}{2} - 2 \sin \frac{\theta}{2} \cos \frac{\theta}{2} + \cos^2 \frac{\theta}{2}} - \sqrt{\sin^2 \frac{\theta}{2} + 2 \sin \frac{\theta}{2} \cos \frac{\theta}{2} + \cos^2 \frac{\theta}{2}}$$

$$= \sqrt{\left(\sin \frac{\theta}{2} - \cos \frac{\theta}{2}\right)^2} - \sqrt{\left(\sin \frac{\theta}{2} + \cos \frac{\theta}{2}\right)^2}$$

$$= \left| \sin \frac{\theta}{2} - \cos \frac{\theta}{2} \right| - \left| \sin \frac{\theta}{2} + \cos \frac{\theta}{2} \right|$$

$\because 0 < \theta < \frac{\pi}{2} \quad \therefore 0 < \frac{\theta}{2} < \frac{\pi}{4} \Rightarrow 0 < \sin \frac{\theta}{2} < \cos \frac{\theta}{2}$

故原式為  $\cos \frac{\theta}{2} - \sin \frac{\theta}{2} - \left(\sin \frac{\theta}{2} + \cos \frac{\theta}{2}\right) = -2 \sin \frac{\theta}{2}$  即為所求

2. 試求  $\cos 20^\circ \cos 40^\circ \cos 60^\circ \cos 80^\circ$  之值。(25 分)

解：  $\cos 20^\circ \cos 40^\circ \cos 60^\circ \cos 80^\circ$

$$= \frac{1}{2 \sin 20^\circ} (2 \sin 20^\circ \cos 20^\circ) \cos 40^\circ \times \frac{1}{2} \times \cos 80^\circ$$

$$= \frac{1}{4 \sin 20^\circ} \times \sin 40^\circ \cos 40^\circ \cos 80^\circ = \frac{1}{8 \sin 20^\circ} (2 \sin 40^\circ \cos 40^\circ) \cos 80^\circ$$

$$= \frac{1}{8 \sin 20^\circ} \times \sin 80^\circ \cos 80^\circ = \frac{1}{16 \sin 20^\circ} (2 \sin 80^\circ \cos 80^\circ)$$

$$= \frac{1}{16 \sin 20^\circ} \times \sin 160^\circ = \frac{1}{16 \sin 20^\circ} \times \sin 20^\circ = \frac{1}{16}$$
 即為所求

3. 試化簡  $\frac{\cos 9\theta - \cos 5\theta + \cos \theta}{\sin 9\theta - \sin 5\theta + \sin \theta}$ . (25 分)

$$\begin{aligned}\text{解：} & \frac{\cos 9\theta - \cos 5\theta + \cos \theta}{\sin 9\theta - \sin 5\theta + \sin \theta} = \frac{(\cos 9\theta + \cos \theta) - \cos 5\theta}{(\sin 9\theta + \sin \theta) - \sin 5\theta} \\ & = \frac{2 \cos 5\theta \cos 4\theta - \cos 5\theta}{2 \sin 5\theta \cos 4\theta - \sin 5\theta} = \frac{\cos 5\theta (2 \cos 4\theta - 1)}{\sin 5\theta (2 \cos 4\theta - 1)} \\ & = \frac{\cos 5\theta}{\sin 5\theta} = \cot 5\theta \text{ 即為所求}\end{aligned}$$

4. 試求：(1)  $\cos 25^\circ + \cos 95^\circ + \cos 145^\circ = ?$  (10 分)

(2)  $\sin 25^\circ - \sin 85^\circ + \sin 145^\circ - \sin 150^\circ = ?$  (15 分)

$$\begin{aligned}\text{解：} & (1) \cos 25^\circ + \cos 95^\circ + \cos 145^\circ = (\cos 95^\circ + \cos 25^\circ) + \cos 145^\circ \\ & = 2 \cos 60^\circ \cos 35^\circ + \cos (180^\circ - 35^\circ) = 2 \times \frac{1}{2} \times \cos 35^\circ - \cos 35^\circ \\ & = \cos 35^\circ - \cos 35^\circ = 0 \text{ 即為所求} \\ & (2) \sin 25^\circ - \sin 85^\circ + \sin 145^\circ - \sin 150^\circ \\ & = (\sin 145^\circ + \sin 25^\circ) - \sin 85^\circ - \sin (180^\circ - 30^\circ) \\ & = 2 \sin 85^\circ \cos 60^\circ - \sin 85^\circ - \sin 30^\circ = 2 \times \frac{1}{2} \times \sin 85^\circ - \sin 85^\circ - \frac{1}{2} \\ & = \sin 85^\circ - \sin 85^\circ - \frac{1}{2} = -\frac{1}{2} \text{ 即為所求}\end{aligned}$$