高雄市明誠中學 高一數學平時測驗　　日期：99.07.23					
範圍	3-3(2) 積與和差的互換	班級		姓名	
		座號			

一、單選題(每題 5 分)

1.△ABC 中，已知 $\cos B\cos C=\sin^2\dfrac{A}{2}$，則△$ABC$ 的形狀必爲

(A)等腰三角形　(B)正三角形　(C)直角三角形　(D)鈍角三角形　(E)銳角三角形

【解答】(A)

【詳解】

$\because\ \cos B\cos C=\sin^2\dfrac{A}{2}\ \Rightarrow\ 2\cos B\cos C=2\sin^2\dfrac{A}{2}$

$\Rightarrow\ \cos(B+C)+\cos(B-C)=2(\dfrac{1-\cos A}{2})$

$\Rightarrow\ -\cos A+\cos(B-C)=1-\cos A\Rightarrow\ \cos(B-C)=1$

$\therefore\ B-C=0\ \Rightarrow\ B=C\ \therefore$ △ABC 爲等腰三角形

2.$\sin20°\cos70°+\sin10°\sin50°$的值爲(A)$\dfrac{3}{4}$　(B)$\dfrac{1}{4}$　(C) 0　(D)$-\dfrac{1}{4}$　(E)$-\dfrac{3}{4}$

【解答】(B)

【詳解】

$\sin20°\cos70°+\sin10°\sin50°=\dfrac{1}{2}[\sin90°+\sin(-50°)]+(\dfrac{-1}{2})(\cos60°-\cos40°)$

$=\dfrac{1}{2}(1-\sin50°)-\dfrac{1}{2}(\dfrac{1}{2}-\cos40°)=\dfrac{1}{2}-\dfrac{1}{2}\sin50°-\dfrac{1}{4}+\dfrac{1}{2}\sin50°=\dfrac{1}{4}$

3.若 $A+B+C=180°$，則 $\sin2A+\sin2B+\sin2C=$

(A) $4\sin A\sin B\cos C$　(B) $4\cos A\cos B\cos C$　(C) $4\sin A\cos B\cos C$　(D) $4\cos A\cos B\sin C$　(E) $4\sin A\sin B\sin C$

【解答】(E)

【詳解】

$\sin2A+\sin2B+\sin2C=2\sin(A+B)\cos(A-B)+2\sin C\cos C$

$=2\sin C\cos(A-B)+2\sin C[-\cos(A+B)]=-2\sin C[\cos(A+B)-\cos(A-B)]$

$=-2\sin C(-2\sin A\sin B)=4\sin A\sin B\sin C$

4.設 $x+y=\dfrac{\pi}{6}$，則 $\cos^2x+\sin^2y$ 的最大值爲(A)$\dfrac{3}{2}$　(B)$\dfrac{\sqrt{3}}{2}$　(C)$\dfrac{\sqrt{2}}{2}$　(D)$\sqrt{3}$　(E) 2

【解答】(A)

【詳解】

$\cos^2 x + \sin^2 y = \frac{1}{2}(1 + \cos 2x) + \frac{1}{2}(1 - \cos 2y) = 1 + \frac{1}{2}(\cos 2x - \cos 2y)$

$= 1 + \frac{1}{2}[-2\sin(x+y)\sin(x-y)] = 1 - \frac{1}{2}\sin(x-y) = 1 - \frac{1}{2}\sin(2x - \frac{\pi}{6})$

$\therefore$ 當 $\sin(2x - \frac{\pi}{6}) = -1$ 時，最大值爲 $\frac{3}{2}$

5.$\sin 52.5° + \sin 7.5° =$ (A) $\sin 22.5°$ (B) $\cos 22.5°$ (C) $\sin 11.25°$ (D) $\cos 11.25°$ (E) $\cos 5.625°$

【解答】(B)

【詳解】

$\sin 52.5° + \sin 7.5° = 2\sin\frac{52.5° + 7.5°}{2}\cos\frac{52.5° - 7.5°}{2} = 2\sin 30°\cos 22.5° = \cos 22.5°$

二、多重選擇題(每題 10 分)

1.下列敘述，何者正確？

(A) $\cos 10°\cos 50°\cos 70° = \frac{1}{8}$ (B) $\cot\frac{\pi}{9}\ \cot\frac{2\pi}{9}\ \cot\frac{4\pi}{9} = \frac{1}{\sqrt{3}}$

(C) $\tan\frac{\pi}{18}\ \tan\frac{5\pi}{18}\ \tan\frac{7\pi}{18} = \frac{1}{\sqrt{3}}$ (D) $\sec\frac{\pi}{9}\sec\frac{2\pi}{9}\sec\frac{4\pi}{9} = 8$ (E) $\csc\frac{\pi}{18}\ \csc\frac{5\pi}{18}\ \csc\frac{7\pi}{18} = 8$

【解答】(B)(C)(D)(E)

【詳解】

$\because \cos 20°\cos 40°\cos 80° = \frac{8\sin 20°\cos 20°\cos 40°\cos 80°}{8\sin 20°} = \frac{\sin 160°}{8\sin 20°} = \frac{\sin 20°}{8\sin 20°} = \frac{1}{8}$

又 $\sin 20°\sin 40°\sin 80° = -\frac{1}{2}\sin 40°(-2\sin 20°\sin 80°)$

$= -\frac{1}{2}\sin 40°(\cos 100° - \cos 60°) = -\frac{1}{4}(2\sin 40°\cos 100°) + \frac{1}{4}\sin 40°$

$= -\frac{1}{4}[\sin 140° + \sin(-60°)] + \frac{1}{4}\sin 40° = -\frac{1}{4}\sin 40° + \frac{1}{4}\sin 60° + \frac{1}{4}\sin 40° = \frac{\sqrt{3}}{8}$

(A) $\cos 10°\cos 50°\cos 70° = \sin 20°\sin 40°\sin 80° = \frac{\sqrt{3}}{8}$

(B) $\cot\frac{\pi}{9}\cot\frac{2\pi}{9}\cot\frac{4\pi}{9} = \cot 20°\cot 40°\cot 80° = \frac{\cos 20°\cos 40°\cos 80°}{\sin 20°\sin 40°\sin 80°} = \frac{\frac{1}{8}}{\frac{\sqrt{3}}{8}} = \frac{1}{\sqrt{3}}$

(C) $\tan\frac{\pi}{18}\tan\frac{5\pi}{18}\tan\frac{7\pi}{18} = \tan 10°\tan 50°\tan 70° = \cot 20°\cot 40°\cot 80° = \frac{1}{\sqrt{3}}$

(D) $\sec\frac{\pi}{9}\sec\frac{2\pi}{9}\sec\frac{4\pi}{9} = \sec 20°\sec 40°\sec 80° = \frac{1}{\cos 20°\cos 40°\cos 80°} = 8$

(E) $\csc\dfrac{\pi}{18}\csc\dfrac{5\pi}{18}\csc\dfrac{7\pi}{18}=\csc10°\csc50°\csc70°=\sec20°\sec40°\sec80°=8$

2.設 $x+y=\dfrac{\pi}{3}$，下列何者正確？

(A) $\cos x+\cos y$ 之最大值為 $\sqrt{3}$　(B) $\sin x+\sin y$ 之最小值為 $-\sqrt{3}$

(C) $\cos x\cos y$ 之最大值為 $\dfrac{3}{4}$　(D) $\sin x\sin y$ 之最小值為 $-\dfrac{3}{4}$　(E) $\cos^2x+\cos^2y$ 之最大值為 $\dfrac{3}{2}$

【解答】(A)(C)(D)(E)

【詳解】

(A) $\cos x+\cos y=2\cos\dfrac{x+y}{2}\cos\dfrac{x-y}{2}=2\cos\dfrac{\pi}{6}\cos\dfrac{x-y}{2}=\sqrt{3}\cos\dfrac{x-y}{2}$

(B) $\sin x+\sin y=2\sin\dfrac{x+y}{2}\cos\dfrac{x-y}{2}=2\sin\dfrac{\pi}{6}\cos\dfrac{x-y}{2}=\cos\dfrac{x-y}{2}$

(C) $\cos x\cos y=\dfrac{1}{2}[\cos(x+y)+\cos(x-y)]=\dfrac{1}{2}\cos\dfrac{\pi}{3}+\dfrac{1}{2}\cos(x-y)=\dfrac{1}{4}+\dfrac{1}{2}\cos(x-y)\Rightarrow \text{Max}=\dfrac{3}{4}$

(D) $\sin x\sin y=-\dfrac{1}{2}[\cos(x+y)-\cos(x-y)]=-\dfrac{1}{2}\cos\dfrac{\pi}{3}+\dfrac{1}{2}\cos(x-y)=-\dfrac{1}{4}+\dfrac{1}{2}\cos(x-y)\Rightarrow \min$

$=-\dfrac{3}{4}$

(E) $\cos^2x+\cos^2y=\dfrac{1+\cos2x}{2}+\dfrac{1+\cos2y}{2}=1+\dfrac{1}{2}(\cos2x+\cos2y)$

$=1+\cos(x+y)\cos(x-y)=1+\dfrac{1}{2}\cos(x-y)\Rightarrow \text{Max}=\dfrac{3}{2}$

三、填充題(每題 10 分)

1.$\sin80°-\sin40°-\sin20°=$ ＿＿＿＿＿＿＿＿＿＿。

【解答】0

【詳解】

$\sin80°-\sin40°-\sin20°=(\sin80°-\sin40°)-\sin20°=2\cos\dfrac{80°+40°}{2}\sin\dfrac{80°-40°}{2}-\sin20°$

$=2\cos60°\sin20°-\sin20°=2\cdot\dfrac{1}{2}\cdot\sin20°-\sin20°=\sin20°-\sin20°=0$

2.$f(x)=\sin x\sin(60°-x)\sin(60°+x)$的最大值為＿＿＿＿＿＿＿＿＿＿。

【解答】$\dfrac{1}{4}$

【詳解】

$$f(x)=\sin x\sin(60°-x)\sin(60°+x)=-\frac{1}{2}\sin x[-2\sin(60°-x)\sin(60°+x)]$$

$$=-\frac{1}{2}\sin x(\cos120°-\cos2x)=\frac{1}{4}\sin x+\frac{1}{2}\sin x\cos2x$$

$$=\frac{1}{4}\sin x+\frac{1}{2}\sin x(1-2\sin^2 x)=\frac{3}{4}\sin x-\sin^3 x=\frac{1}{4}(3\sin x-4\sin^3 x)=\frac{1}{4}\sin3x$$

∴　當 $\sin3x=1$ 時，$f(x)$有最大值爲$\frac{1}{4}$

【註】

(1) $\sin x\sin(60°-x)\sin(60°+x)=\frac{1}{4}\sin3x$

(2) $\cos x\cos(60°-x)\cos(60°+x)=\frac{1}{4}\cos3x$

(3)利用(1)(2) $\sin20°\sin40°\sin80°=\frac{\sqrt{3}}{8}$，$\cos20°\cos40°\cos80°=\frac{1}{8}$

3.$\sin^2 10°+\cos^2 20°-\sin10°\cos20°=$ ________。

【解答】$\frac{3}{4}$

【詳解】

$$\sin^2 10°+\cos^2 20°-\sin10°\cos20°=\frac{1-\cos20°}{2}+\frac{1+\cos40°}{2}-\sin10°\cos20°$$

$$=1+\frac{1}{2}(\cos40°-\cos20°)-\frac{1}{2}(2\sin10°\cos20°)$$

$$=1+\frac{1}{2}(-2\sin30°\sin10°)-\frac{1}{2}(\sin30°-\sin10°)$$

$$=1-\frac{1}{2}\sin10°-\frac{1}{2}(\frac{1}{2}-\sin10°)$$

$$=1-\frac{1}{4}=\frac{3}{4}$$

4.$\triangle ABC$ 中，$\cos A=-\frac{3}{5}$，$\cos B=\frac{12}{13}$，則 $\cos C=$ ________。

【解答】$\frac{56}{65}$

【詳解】$\cos A=-\frac{3}{5}\ \Rightarrow\ \sin A=\frac{4}{5}$，$\cos B=\frac{12}{13}\ \Rightarrow\ \sin B=\frac{5}{13}$

∴　$\cos C=\cos(\pi-(A+B))=-\cos(A+B)$

$$=-(\cos A\cos B-\sin A\sin B)=-(-\frac{3}{5}\cdot\frac{12}{13}-\frac{4}{5}\cdot\frac{5}{13})=\frac{56}{65}$$

5.cos55°sin5° + cos55°sin25° − cos65°sin15°之值爲__________。

【解答】$\dfrac{\sqrt{3}-1}{4}$

【詳解】

原式 $=\dfrac{1}{2}[\sin60° + \sin(-50°)] + \dfrac{1}{2}[\sin80° + \sin(-30°)] - \dfrac{1}{2}[\sin80° + \sin(-50°)]$

$=\dfrac{1}{2}(\dfrac{\sqrt{3}}{2} - \sin50° + \sin80° - \dfrac{1}{2} - \sin80° + \sin50°) = \dfrac{\sqrt{3}-1}{4}$

6.設 $\tan x = \dfrac{\cos 81° + \sin 43°}{\sin 81° - \cos 43°}$。

(1) $0° < x < 180°$時，$x =$ __________。 (2) $180° < x < 360°$時，$x =$ __________。

【解答】(1)73° (2) 253°

【詳解】

$\tan x = \dfrac{\cos 81° + \sin 43°}{\sin 81° - \cos 43°} = \dfrac{\cos 81° + \cos 47°}{\sin 81° - \sin 47°} = \dfrac{2\cos 64°\cos 17°}{2\cos 64°\sin 17°} = \cot 17° = \tan 73°$

(1)∵ $0° < x < 180°$ ∴ $x = 73°$

(2)∵ $180° < x < 360°$ ∴ $x = 253°$

7.在△ABC 中，已知$\angle A：\angle B：\angle C = 1：2：7$ 且 $\sin 18° = \dfrac{\sqrt{5}-1}{4}$，則$\dfrac{c+a}{c-a} =$ __________。

【解答】$\sqrt{5}$

【詳解】

∵ $\angle A：\angle B：\angle C = 1：2：7$ ∴ $\angle A = 18°$，$\angle B = 36°$，$\angle C = 126°$

∵ $\sin 18° = \dfrac{\sqrt{5}-1}{4}$ ∴ $\sin 126° = \sin(180° - 54°)$

$= \sin 54° = \cos 36° = 1 - 2\sin^2 18° = 1 - 2(\dfrac{\sqrt{5}-1}{4})^2 = 1 - \dfrac{6-2\sqrt{5}}{8} = \dfrac{1+\sqrt{5}}{4}$

故由正弦定理：$\dfrac{c+a}{c-a} = \dfrac{\sin C + \sin A}{\sin C - \sin A} = \dfrac{\sin 126° + \sin 18°}{\sin 126° - \sin 18°} = \dfrac{\dfrac{1+\sqrt{5}}{4} + \dfrac{\sqrt{5}-1}{4}}{\dfrac{1+\sqrt{5}}{4} - \dfrac{\sqrt{5}-1}{4}} = \dfrac{\dfrac{\sqrt{5}}{2}}{\dfrac{1}{2}} = \sqrt{5}$

8.$\dfrac{\sin 5° + \sin 20° + \sin 40° + \sin 55°}{\cos 5° + \cos 20° + \cos 40° + \cos 55°} =$ __________。

【解答】$\dfrac{\sqrt{3}}{3}$

【詳解】

$\dfrac{\sin 5° + \sin 20° + \sin 40° + \sin 55°}{\cos 5° + \cos 20° + \cos 40° + \cos 55°} = \dfrac{(\sin 5° + \sin 55°) + (\sin 20° + \sin 40°)}{(\cos 5° + \cos 55°) + (\cos 20° + \cos 40°)}$

$$=\frac{2\sin30^\circ\cos25^\circ+2\sin30^\circ\cos10^\circ}{2\cos30^\circ\cos25^\circ+2\cos30^\circ\cos10^\circ}=\frac{2\cdot\frac{1}{2}\cos25^\circ+2\cdot\frac{1}{2}\cos10^\circ}{2\cdot\frac{\sqrt{3}}{2}\cos25^\circ+2\cdot\frac{\sqrt{3}}{2}\cos10^\circ}$$

$$=\frac{\cos25^\circ+\cos10^\circ}{\sqrt{3}(\cos25^\circ+\cos10^\circ)}=\frac{1}{\sqrt{3}}=\frac{\sqrt{3}}{3}$$

9.設$\theta=\frac{\pi}{21}$，則$\frac{\sin2\theta\sin16\theta}{\cos3\theta-\cos7\theta}=$ ＿＿＿＿＿＿。

【解答】$\frac{1}{2}$

【詳解】∵ $\theta=\frac{\pi}{21}$

$$\therefore\ \frac{\sin2\theta\ \sin16\theta}{\cos3\theta-\cos7\theta}=\frac{\sin2\theta\sin16\theta}{2\sin5\theta\ \sin2\theta}=\frac{\sin16\theta}{2\sin5\theta}=\frac{\sin\frac{16}{21}\pi}{2\sin\frac{5}{21}\pi}=\frac{\sin(\pi-\frac{5}{21}\pi)}{2\sin\frac{5\pi}{21}}=\frac{\sin\frac{5}{21}\pi}{2\sin\frac{5}{21}\pi}=\frac{1}{2}$$

10.$\cos100^\circ\sin50^\circ+\sin50^\circ\cos20^\circ-\cos20^\circ\cos100^\circ=$ ＿＿＿＿＿＿。

【解答】$\frac{3}{4}$

【詳解】

$\cos100^\circ\sin50^\circ+\sin50^\circ\cos20^\circ-\cos20^\circ\cos100^\circ=\sin50^\circ(\cos100^\circ+\cos20^\circ)-\cos20^\circ\cos100^\circ$

$=\sin50^\circ\cdot2\cos60^\circ\cos40^\circ-\frac{1}{2}(2\cos100^\circ\cos20^\circ)=\frac{1}{2}(2\sin50^\circ\cos40^\circ)-\frac{1}{2}(\cos120^\circ+\cos80^\circ)$

$=\frac{1}{2}(\sin90^\circ+\sin10^\circ)-\frac{1}{2}(\cos120^\circ+\cos80^\circ)=\frac{1}{2}(1+\sin10^\circ)-\frac{1}{2}(-\frac{1}{2}+\sin10^\circ)=\frac{1}{2}+\frac{1}{4}=\frac{3}{4}$

11.$\frac{\sin(\theta+\frac{\pi}{4})\sin(\theta-\frac{\pi}{4})}{\sin^2\theta-\cos^2\theta}=$ ＿＿＿＿＿＿。

【解答】$\frac{1}{2}$

【詳解】原式 $=\frac{\sin^2\theta-\sin^2\frac{\pi}{4}}{\sin^2\theta-(1-\sin^2\theta)}=\frac{\sin^2\theta-\frac{1}{2}}{2\sin^2\theta-1}=\frac{1}{2}$

12.$\sin20^\circ\sin35^\circ\sin45^\circ+\cos25^\circ\cos45^\circ\cos80^\circ=$ ＿＿＿＿＿＿。

【解答】$\frac{1}{4}$

【詳解】

$\sin20°\sin35°\sin45° + \cos25°\cos45°\cos80° = \frac{\sqrt{2}}{2}(\sin20°\sin35° + \cos25°\cos80°)$

$= \frac{\sqrt{2}}{2}[\frac{1}{2}(\cos15° - \cos55°) + \frac{1}{2}(\cos105° + \cos55°)] = \frac{\sqrt{2}}{2}(\frac{1}{2}\cos15° + \frac{1}{2}\cos105°)$

$= \frac{\sqrt{2}}{4}(\cos15° + \cos105°) = \frac{\sqrt{2}}{4}(2\cos60°\cos45°) = \frac{\sqrt{2}}{2}\cdot\frac{1}{2}\cdot\frac{\sqrt{2}}{2} = \frac{1}{4}$

13.設 $a = \cos 20°$，$b = \cos 40°$，$c = \cos 80°$，求

(1) $b + c - a =$ ＿＿＿＿＿。 (2) $a^2 + b^2 + c^2 =$ ＿＿＿＿＿。

【解答】(1) 0 (2) $\frac{3}{2}$

【詳解】

(1) $b + c - a = \cos 80° + \cos 40° - \cos 20° = 2\cos 60°\cos20° - \cos 20° = 0$

(2) $a^2 + b^2 + c^2 = \cos^2 20° + \cos^2 40° + \cos^2 80° = \frac{1+\cos40°}{2} + \frac{1+\cos80°}{2} + \frac{1+\cos160°}{2}$

$= \frac{3}{2} + \frac{1}{2}(\cos40° + \cos80° + \cos160°) = \frac{3}{2} + \frac{1}{2}(\cos40° + 2\cos120°\cos40°) = \frac{3}{2}$

14.$\sin^2 27.5° + \sin^2 32.5° + \sin^2 87.5° =$ ＿＿＿＿＿。

【解答】$\frac{3}{2}$

【詳解】

$\sin^2 27.5° + \sin^2 32.5° + \sin^2 87.5° = \frac{1-\cos55°}{2} + \frac{1-\cos65°}{2} + \frac{1-\cos175°}{2}$

$= \frac{3}{2} - \frac{1}{2}[(\cos55° + \cos65°) + \cos175°] = \frac{3}{2} - \frac{1}{2}[2\cos60°\cos5° + (-\cos5°)]$

$= \frac{3}{2} - \frac{1}{2}(2 \times \frac{1}{2} \times \cos5° - \cos5°) = \frac{3}{2} - 0 = \frac{3}{2}$

15.設 $\sin A + \sin B = \frac{1}{2}$，$\cos A + \cos B = \frac{1}{3}$，則 $\tan\frac{1}{2}(A+B) =$ ＿＿＿＿，$\sin(A+B) =$ ＿＿＿＿＿。

【解答】$\frac{3}{2}$，$\frac{12}{13}$

【詳解】

二式相除，得 $\frac{\sin A + \sin B}{\cos A + \cos B} = \frac{2\sin\frac{A+B}{2}\cos\frac{A-B}{2}}{2\cos\frac{A+B}{2}\cos\frac{A-B}{2}} = \tan\frac{A+B}{2}$

$$\tan\frac{1}{2}(A+B)=\frac{\sin A+\sin B}{\cos A+\cos B}=\frac{\frac{1}{2}}{\frac{1}{3}}=\frac{3}{2}$$

$$\sin(A+B)=\frac{2\tan\frac{1}{2}(A+B)}{1+\tan^2\frac{1}{2}(A+B)}=\frac{2\cdot\frac{3}{2}}{1+(\frac{3}{2})^2}=\frac{12}{13}$$

16.$\sin\theta$，$\cos\theta$ 為 $x^2+px+q=0$ 之二根，試以 p，q 表 $2\sin^2\frac{\theta}{2}(\cos\frac{\theta}{2}-\sin\frac{\theta}{2})^2=$ ＿＿＿＿＿。

【解答】$1+p+q$

【詳解】

∵ $\sin\theta$，$\cos\theta$ 為 $x^2+px+q=0$ 之二根 ∴ $\sin\theta+\cos\theta=-p$，$\sin\theta\cos\theta=q$

$2\sin^2\frac{\theta}{2}(\cos\frac{\theta}{2}-\sin\frac{\theta}{2})^2=2\cdot\frac{1-\cos\theta}{2}\cdot(1-2\sin\frac{\theta}{2}\cos\frac{\theta}{2})$

$=(1-\cos\theta)(1-\sin\theta)=1-(\sin\theta+\cos\theta)+\sin\theta\cos\theta=1+p+q$

17.$\cos\frac{\pi}{11}+\cos\frac{3\pi}{11}+\cos\frac{5\pi}{11}+\cos\frac{7\pi}{11}+\cos\frac{9\pi}{11}=$ ＿＿＿＿＿。

【解答】$\frac{1}{2}$

【詳解】

令 $p=\cos\frac{\pi}{11}+\cos\frac{3\pi}{11}+\cos\frac{5\pi}{11}+\cos\frac{7\pi}{11}+\cos\frac{9\pi}{11}$

$\therefore 2p\sin\frac{\pi}{11}=2\cos\frac{\pi}{11}\sin\frac{\pi}{11}+2\cos\frac{3\pi}{11}\sin\frac{\pi}{11}+2\cos\frac{5\pi}{11}\sin\frac{\pi}{11}+2\cos\frac{7\pi}{11}\sin\frac{\pi}{11}+2\cos\frac{9\pi}{11}\sin\frac{\pi}{11}$

$=\sin\frac{2\pi}{11}+(\sin\frac{4\pi}{11}-\sin\frac{2\pi}{11})+(\sin\frac{6\pi}{11}-\sin\frac{4\pi}{11})+(\sin\frac{8\pi}{11}-\sin\frac{6\pi}{11})+(\sin\frac{10\pi}{11}-\sin\frac{8\pi}{11})$

$=\sin\frac{10\pi}{11}=\sin\frac{\pi}{11}$，故 $p=\frac{1}{2}$

18.若 $\begin{cases}\cos\alpha+\cos\beta=\frac{1}{2}\\ \sin\alpha+\sin\beta=\frac{1}{3}\end{cases}$，則 $\sin(\alpha+\beta)=$ ＿＿＿＿＿。

【解答】$\frac{12}{13}$

【詳解】$\begin{cases}\cos\alpha+\cos\beta=\dfrac{1}{2}\\ \sin\alpha+\sin\beta=\dfrac{1}{3}\end{cases}$ $\Rightarrow$ $\begin{cases}2\cos\dfrac{\alpha+\beta}{2}\cos\dfrac{\alpha-\beta}{2}=\dfrac{1}{2}\cdots\cdots ①\\ 2\sin\dfrac{\alpha+\beta}{2}\cos\dfrac{\alpha-\beta}{2}=\dfrac{1}{3}\cdots\cdots ②\end{cases}$

$\dfrac{②}{①}$ $\Rightarrow$ $\tan(\dfrac{\alpha+\beta}{2})=\dfrac{2}{3}$，$\sin(\alpha+\beta)=\dfrac{2\tan(\dfrac{\alpha+\beta}{2})}{1+\tan^2(\dfrac{\alpha+\beta}{2})}=\dfrac{12}{13}$

19.設 $\sin\alpha+\sin\beta+\sin\gamma=0$ 且 $\cos\alpha+\cos\beta+\cos\gamma=0$，則

(1) $\cos(\alpha-\beta)=$ ＿＿＿＿＿。 (2) $\sin2\alpha+\sin2\beta+\sin2\gamma=$ ＿＿＿＿＿。

(3) $\cos2\alpha+\cos2\beta+\cos2\gamma=$ ＿＿＿＿＿。 (4) $\cos^2\alpha+\cos^2\beta+\cos^2\gamma=$ ＿＿＿＿＿。

(5) $\sin^2\alpha+\sin^2\beta+\sin^2\gamma=$ ＿＿＿＿＿。

【解答】(1) $-\dfrac{1}{2}$ (2) 0 (3) 0 (4) $\dfrac{3}{2}$ (5) $\dfrac{3}{2}$

【詳解】

(1) $\begin{cases}\sin\alpha+\sin\beta=-\sin\gamma\cdots\cdots ①\\ \cos\alpha+\cos\beta=-\cos\gamma\cdots\cdots ②\end{cases}$

$①^2+②^2$ 得 $2+2(\cos\alpha\cos\beta+\sin\alpha\sin\beta)=1$ $\Rightarrow$ $2+2\cos(\alpha-\beta)=1$ $\therefore$ $\cos(\alpha-\beta)=-\dfrac{1}{2}$

$①\times②$得$(\sin\alpha+\sin\beta)(\cos\alpha+\cos\beta)=\sin\gamma\cos\gamma$

$\Rightarrow$ $\sin\alpha\cos\alpha+\sin\beta\cos\beta+\sin\alpha\cos\beta+\cos\alpha\sin\beta=\sin\gamma\cos\gamma$

$\Rightarrow$ $\dfrac{1}{2}\sin2\alpha+\dfrac{1}{2}\sin2\beta+\sin(\alpha+\beta)=\dfrac{1}{2}\sin2\gamma$ $\Rightarrow$ $\sin(\alpha+\beta)\cos(\alpha-\beta)+\sin(\alpha+\beta)=\dfrac{1}{2}\sin2\gamma$

$\Rightarrow$ $\sin(\alpha+\beta)(-\dfrac{1}{2})+\sin(\alpha+\beta)=\dfrac{1}{2}\sin2\gamma$

$\therefore$ $\sin(\alpha+\beta)=\sin2\gamma$

(2) $\sin2\alpha+\sin2\beta+\sin2\gamma=2\sin(\alpha+\beta)\cos(\alpha-\beta)+\sin2\gamma$

$=-\sin(\alpha+\beta)+\sin2\gamma=-\sin2\gamma+\sin2\gamma=0$

(3)$②^2-①^2$ $(\cos\alpha+\cos\beta)^2-(\sin\alpha+\sin\beta)^2=\cos^2\gamma-\sin^2\gamma$

$\Rightarrow$ $\cos^2\alpha+2\cos\alpha\cos\beta+\cos^2\beta-\sin^2\alpha-2\sin\alpha\sin\beta-\sin^2\beta=\cos^2\gamma-\sin^2\gamma$

$\Rightarrow$ $\cos2\alpha+\cos2\beta+2\cos(\alpha+\beta)=\cos2\gamma$

$\Rightarrow$ $2\cos(\alpha+\beta)\cos(\alpha-\beta)+2\cos(\alpha+\beta)=\cos2\gamma$

$\Rightarrow$ $-\cos(\alpha+\beta)+2\cos(\alpha+\beta)=\cos2\gamma$

$\therefore$ $\cos(\alpha+\beta)=\cos2\gamma$

$\cos2\alpha+\cos2\beta+\cos2\gamma=2\cos(\alpha+\beta)\cos(\alpha-\beta)+\cos2\gamma$

$=-\cos(\alpha+\beta)+\cos2\gamma=-\cos2\gamma+\cos2\gamma=0$

(4) $\cos^2\alpha+\cos^2\beta+\cos^2\gamma=\dfrac{1+\cos2\alpha}{2}+\dfrac{1+\cos2\beta}{2}+\dfrac{1+\cos2\gamma}{2}$

$=\dfrac{3}{2}+\dfrac{1}{2}(\cos2\alpha+\cos2\beta+\cos2\gamma)=\dfrac{3}{2}$

(5) $\sin^2\alpha+\sin^2\beta+\sin^2\gamma=\dfrac{1-\cos 2\alpha}{2}+\dfrac{1-\cos 2\beta}{2}+\dfrac{1-\cos 2\gamma}{2}$

$$=\frac{3}{2}-\frac{1}{2}(\cos2\alpha+\cos2\beta+\cos2\gamma)=\frac{3}{2}$$

20.已知 $\sin\alpha+\sin\beta=\dfrac{3}{5}$，$\cos\alpha+\cos\beta=\dfrac{1}{5}$，則

(1) $\tan\dfrac{\alpha+\beta}{2}=$ __________。 (2) $\cos(\alpha+\beta)=$ __________。

【解答】(1)3 (2)$\dfrac{-4}{5}$

【詳解】

(1)$\because$ $\sin\alpha+\sin\beta=\dfrac{3}{5}$ $\therefore$ $2\sin\dfrac{\alpha+\beta}{2}\cos\dfrac{\alpha-\beta}{2}=\dfrac{3}{5}$……①

$\because$ $\cos\alpha+\cos\beta=\dfrac{1}{5}$ $\therefore$ $2\cos\dfrac{\alpha+\beta}{2}\cos\dfrac{\alpha-\beta}{2}=\dfrac{1}{5}$……②

$\dfrac{①}{②}$得 $\dfrac{\sin\dfrac{\alpha+\beta}{2}}{\cos\dfrac{\alpha+\beta}{2}}=\dfrac{\frac{3}{5}}{\frac{1}{5}}$ $\Rightarrow$ $\tan\dfrac{\alpha+\beta}{2}=3$

(2) $\cos(\alpha+\beta)=\dfrac{1-\tan^2\dfrac{\alpha+\beta}{2}}{1+\tan^2\dfrac{\alpha+\beta}{2}}=\dfrac{1-9}{1+9}=-\dfrac{4}{5}$

21.$4\sin20°+\tan20°=$ __________。

【解答】$\sqrt{3}$

【詳解】

$4\sin 20°+\tan 20°=4\sin 20°+\dfrac{\sin 20°}{\cos 20°}=\dfrac{4\sin 20°\cos 20°+\sin 20°}{\cos 20°}=\dfrac{2\sin 40°+\sin 20°}{\cos 20°}$

$=\dfrac{\sin40°+(\sin 40°+\sin 20°)}{\cos 20°}=\dfrac{\sin 40°+2\sin 30°\cos10°}{\cos 20°}$

$=\dfrac{\sin 40°+\cos10°}{\cos 20°}=\dfrac{\cos 50°+\cos 10°}{\cos 20°}=\dfrac{2\cos 30°\cos 20°}{\cos 20°}=2\cos 30°=\sqrt{3}$

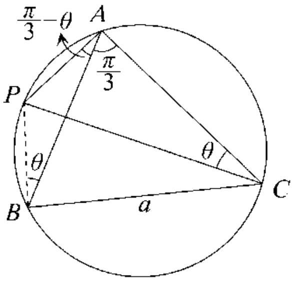
22.設邊長爲 a 的正△ABC 內接於一圓，點 $P\in\overset{\frown}{AB}$ 上，且$\angle ACP=\theta$。

(1) 若以 a 及θ 表△ABP 面積，則其面積爲__________，

(2) △ABP+ △ACP 面積和的最大值爲__________。

【解答】$\dfrac{1}{\sqrt{3}}a^2\sin(\dfrac{\pi}{3}-\theta)\sin\theta$；$\dfrac{1}{2}a^2$

【詳解】



(1)如圖　∵　$\angle ACP=\theta$　∴　$\angle BCP=\frac{\pi}{3}-\theta$，且$\angle ABP=\theta$，$\angle BAP=\frac{\pi}{3}-\theta$

∵　$\triangle ABP$ 之面積 $=\frac{1}{2}\overline{AB}\cdot\overline{BP}\sin\theta$且 $\overline{AB}=a=\overline{BC}$

於$\triangle ABP$中，由正弦定理$\frac{\overline{BP}}{\sin(\frac{\pi}{3}-\theta)}=\frac{\overline{AB}}{\sin\angle BPA}=\frac{a}{\sin\frac{2\pi}{3}}=\frac{a}{\frac{\sqrt{3}}{2}}=\frac{2a}{\sqrt{3}}$，∴　$\overline{BP}=\frac{2a}{\sqrt{3}}\sin(\frac{\pi}{3}-\theta)$

故$\triangle ABP$ 之面積 $=\frac{1}{2}\cdot a\cdot\frac{2a}{\sqrt{3}}\sin(\frac{\pi}{3}-\theta)\cdot\sin\theta=\frac{1}{\sqrt{3}}a^2\sin(\frac{\pi}{3}-\theta)\sin\theta$

(2)　∵　$\frac{\overline{AP}}{\sin\theta}=\frac{\overline{AB}}{\sin\frac{2\pi}{3}}=\frac{a}{\frac{\sqrt{3}}{2}}=\frac{2a}{\sqrt{3}}$　∴　$\overline{AP}=\frac{2a}{\sqrt{3}}\sin\theta$

故$\triangle APC$ 之面積 $=\frac{1}{2}\cdot\overline{AP}\cdot\overline{AC}\sin(\frac{2\pi}{3}-\theta)$

$=\frac{1}{2}\cdot\frac{2a}{\sqrt{3}}\sin\theta\cdot a\sin(\frac{2\pi}{3}-\theta)=\frac{a^2}{\sqrt{3}}\sin\theta\cdot\sin(\frac{2\pi}{3}-\theta)$

∴　$\triangle ABP$ 之面積 ＋ $\triangle APC$ 之面積 $=\frac{1}{\sqrt{3}}a^2\sin\theta\sin(\frac{\pi}{3}-\theta)+\frac{1}{\sqrt{3}}a^2\sin\theta\sin(\frac{2\pi}{3}-\theta)$

$=\frac{a^2}{\sqrt{3}}\sin\theta[\sin(\frac{\pi}{3}-\theta)+\sin(\frac{2\pi}{3}-\theta)]=\frac{a^2}{\sqrt{3}}\sin\theta\cdot 2\sin(\frac{\pi}{2}-\theta)\cos\frac{\pi}{6}$

$=\frac{a^2}{\sqrt{3}}\sin\theta\cdot 2\cos\theta\cdot\frac{\sqrt{3}}{2}=\frac{1}{2}a^2\sin 2\theta$

當 $\sin 2\theta=1$ 時，$\triangle ABP+\triangle ACP$ 面積和的最大值爲$\frac{1}{2}a^2$

23.設α，β 不同界，已知α，β 爲方程式 $\sin x-\sqrt{3}\cos x=1$ 的兩個根，則 $\tan\frac{\alpha+\beta}{2}$之值爲______。

【解答】$-\frac{\sqrt{3}}{3}$

【詳解】

(1) α，β 爲 $\sin x-\sqrt{3}\cos x=1$ 的兩個根

$$\therefore\quad \sin\alpha-\sqrt{3}\cos\alpha=1$$
$$-)\quad \sin\beta-\sqrt{3}\cos\beta=1$$
$$(\sin\alpha-\sin\beta)-\sqrt{3}(\cos\alpha-\cos\beta)=0$$

$$2\cos\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2}=\sqrt{3}\left(-2\sin\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2}\right)$$

(2)但α，β 不同界　$\therefore$　$\alpha-\beta\neq 2k\pi$，$k\in Z\Rightarrow$　$\frac{\alpha-\beta}{2}\neq k\pi$，$k\in Z$　$\Rightarrow$　$\sin\frac{\alpha-\beta}{2}\neq 0$

(3)由(1)(2)得 $\cos\frac{\alpha+\beta}{2}=-\sqrt{3}\sin\frac{\alpha+\beta}{2}$　$\Rightarrow$　$\tan\frac{\alpha+\beta}{2}=-\frac{1}{\sqrt{3}}=-\frac{\sqrt{3}}{3}$

24.設 $\sin\alpha+\sin\beta=\frac{1}{2}$，$\cos\alpha+\cos\beta=\frac{1}{3}$，則：

(1) $\cos(\alpha-\beta)=$ ＿＿＿＿＿＿。　(2) $\cos(\alpha+\beta)=$ ＿＿＿＿＿＿。

【解答】(1) $-\frac{59}{72}$　(2) $-\frac{5}{13}$

【詳解】

(1)$\begin{cases}\sin\alpha+\sin\beta=\frac{1}{2}\\ \cos\alpha+\cos\beta=\frac{1}{3}\end{cases}$，平方得$\begin{cases}1+2\sin\alpha\sin\beta=\frac{1}{4}\\ 1+2\cos\alpha\cos\beta=\frac{1}{9}\end{cases}$　$\Rightarrow$　$\begin{cases}\sin\alpha\sin\beta=-\frac{3}{8}\cdots\cdots ①\\ \cos\alpha\cos\beta=-\frac{4}{9}\cdots\cdots ②\end{cases}$

由① + ②得 $\sin\alpha\sin\beta+\cos\alpha\cos\beta=-\frac{59}{72}$　$\Rightarrow$　$\cos(\alpha-\beta)=-\frac{59}{72}$

(2)$\begin{cases}\sin\alpha+\sin\beta=\frac{1}{2}\\ \cos\alpha+\cos\beta=\frac{1}{3}\end{cases}$　$\Rightarrow$　$\begin{cases}2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}=\frac{1}{2}\cdots\cdots ①\\ 2\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}=\frac{1}{3}\cdots\cdots ②\end{cases}$

由$\frac{①}{②}$得 $\tan\frac{\alpha+\beta}{2}=\frac{3}{2}\Rightarrow$　$\cos(\alpha+\beta)=\dfrac{1-\tan^2\frac{\alpha+\beta}{2}}{1+\tan^2\frac{\alpha+\beta}{2}}=\dfrac{1-(\frac{3}{2})^2}{1+(\frac{3}{2})^2}=-\frac{5}{13}$

25.$f(\theta)=\dfrac{\sin 2\theta+\sin 4\theta+\sin 6\theta+\sin 8\theta}{\cos 2\theta+\cos 4\theta+\cos 6\theta+\cos 8\theta}$，

(1)若 $f(\theta)=\tan k\theta$，則 $k=$ ＿＿＿＿＿＿。　(2) $f(24°)=$ ＿＿＿＿＿＿。

【解答】(1) 5　(2) $-\sqrt{3}$

【詳解】

$$(1)\ f(\theta)=\frac{(\sin 2\theta+\sin 6\theta)+(\sin 4\theta+\sin 8\theta)}{(\cos 2\theta+\cos 6\theta)+(\cos 4\theta+\cos 8\theta)}=\frac{2\sin 4\theta\cos 2\theta+2\sin 6\theta\cos 2\theta}{2\cos 4\theta\cos 2\theta+2\cos 6\theta\cos 2\theta}$$
$$=\frac{2(\sin 4\theta+\sin 6\theta)}{2(\cos 4\theta+\cos 6\theta)}=\frac{4\sin 5\theta\cos\theta}{4\cos 5\theta\cos\theta}=\tan 5\theta\qquad\therefore\quad k=5$$

(2)由(1)可知 $f(24°)=\tan(5\cdot 24°)=\tan120°=-\sqrt{3}$

26. $\cos^2\theta+\cos^2(\theta+\frac{\pi}{5})+\cos^2(\theta+\frac{2\pi}{5})+\cos^2(\theta+\frac{3\pi}{5})+\cos^2(\theta+\frac{4\pi}{5})=$ ________。

【解答】 $\frac{5}{2}$

【詳解】

$\cos^2\theta+\cos^2(\theta+\frac{\pi}{5})+\cos^2(\theta+\frac{2\pi}{5})+\cos^2(\theta+\frac{3\pi}{5})+\cos^2(\theta+\frac{4\pi}{5})$

$=\cos^2\theta+\cos^2(\theta+\frac{\pi}{5})+\cos^2(\theta+\frac{2\pi}{5})+\cos^2(\theta+\pi-\frac{2\pi}{5})+\cos^2(\theta+\pi-\frac{\pi}{5})$

$=\cos^2\theta+\cos^2(\theta+\frac{\pi}{5})+\cos^2(\theta+\frac{2\pi}{5})+\cos^2(\theta-\frac{2\pi}{5})+\cos^2(\theta-\frac{\pi}{5})$

$=\frac{1+\cos2\theta}{2}+\frac{1+\cos(2\theta+\frac{2\pi}{5})}{2}+\frac{1+\cos(2\theta+\frac{4\pi}{5})}{2}+\frac{1+\cos(2\theta-\frac{4\pi}{5})}{2}+\frac{1+\cos(2\theta-\frac{2\pi}{5})}{2}$

$=\frac{5}{2}+\frac{1}{2}[\cos2\theta+(\cos(2\theta+\frac{2\pi}{5})+\cos(2\theta-\frac{2\pi}{5}))+(\cos(2\theta+\frac{4\pi}{5})+\cos(2\theta-\frac{4\pi}{5}))]$

$=\frac{5}{2}+\frac{1}{2}(\cos2\theta+2\cos2\theta\cdot\cos\frac{2\pi}{5}+2\cos2\theta\cdot\cos\frac{4\pi}{5})$

$=\frac{5}{2}+\frac{1}{2}[\cos2\theta+2\cos2\theta(\cos\frac{2\pi}{5}+\cos\frac{4\pi}{5})]=\frac{5}{2}+\frac{1}{2}[\cos2\theta+2\cos2\theta(\cos72°+\cos144°)]$

$=\frac{5}{2}+\frac{1}{2}[\cos2\theta+2\cos2\theta(\sin18°-\cos36°)]=\frac{5}{2}+\frac{1}{2}[\cos2\theta+2\cos2\theta(\frac{\sqrt{5}-1}{4}-\frac{\sqrt{5}+1}{4})]$

$=\frac{5}{2}+\frac{1}{2}[\cos2\theta+2\cos2\theta(-\frac{1}{2})]=\frac{5}{2}+\frac{1}{2}(\cos2\theta-\cos2\theta)=\frac{5}{2}+0=\frac{5}{2}$