

計算題 (共 100 分)

1. 設 $\frac{3\pi}{2} < \alpha < 2\pi$, $\frac{\pi}{2} < \beta < \pi$, 試求 $\cos\alpha = \frac{3}{5}$, $\sin\beta = \frac{12}{13}$, 試求 $\sin(\alpha - \beta)$ 之值。(20 分)

解：由 $\frac{3\pi}{2} < \alpha < 2\pi$, $\cos\alpha = \frac{3}{5} \Rightarrow \sin\alpha = -\sqrt{1 - \cos^2\alpha} = -\sqrt{1 - (\frac{3}{5})^2} = -\frac{4}{5}$

又 $\frac{\pi}{2} < \beta < \pi$, $\sin\beta = \frac{12}{13} \Rightarrow \cos\beta = -\sqrt{1 - \sin^2\beta} = -\sqrt{1 - (\frac{12}{13})^2} = -\frac{5}{13}$

$\therefore \sin(\alpha - \beta) = \sin\alpha \cos\beta - \cos\alpha \sin\beta = (-\frac{4}{5}) \times (-\frac{5}{13}) - \frac{3}{5} \times \frac{12}{13}$

$= \frac{20}{65} - \frac{36}{65} = -\frac{16}{65}$ 即為所求

2. 試求 $\tan 22^\circ + \tan 23^\circ + \tan 22^\circ \tan 23^\circ$ 之值。(20 分)

解：由 $\tan(22^\circ + 23^\circ) = \frac{\tan 22^\circ + \tan 23^\circ}{1 - \tan 22^\circ \tan 23^\circ}$

$\Rightarrow \frac{\tan 22^\circ + \tan 23^\circ}{1 - \tan 22^\circ \tan 23^\circ} = \tan 45^\circ = 1$

$\Rightarrow \tan 22^\circ + \tan 23^\circ = 1 - \tan 22^\circ \tan 23^\circ$

$\therefore \tan 22^\circ + \tan 23^\circ + \tan 22^\circ \tan 23^\circ = 1$ 即為所求

3. 設 A, B, C 均為銳角且 $\tan A = 4, \tan B = 2, \tan C = 13$, 試求 $\angle A + \angle B + \angle C$ 之值. (20分)

解：由 $\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B} = \frac{4+2}{1-4 \times 2} = \frac{6}{1-8} = -\frac{6}{7} < 0$

$\Rightarrow \angle A + \angle B > 90^\circ \dots\dots\dots (*)$

$\tan(A+B+C) = \tan[(A+B)+C]$

$$= \frac{-\frac{6}{7} + 13}{1 - (-\frac{6}{7}) \times 13} = \frac{\frac{85}{7}}{\frac{85}{7}} = 1 = \tan \frac{\pi}{4} = \tan \frac{5\pi}{4} > 0$$

$\Rightarrow \angle A + \angle B + \angle C$ 為第一或第三象限角

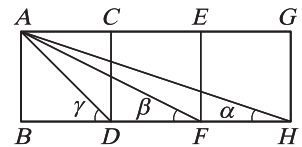
但由 (*) 知 $\angle A + \angle B + \angle C$ 為第二象限角

故 $\angle A + \angle B + \angle C = \pi + \frac{\pi}{4} = \frac{5\pi}{4} = 225^\circ$ 即為所求

4. 右圖中由三個正方形緊鄰接成一個矩形，試求：

(1) $\alpha + \beta + \gamma$ 之值. (20分)

(2) $\tan \angle FAH$ 之值. (20分)



解：(1) 由題圖可得 $\tan \alpha = \frac{1}{3}, \tan \beta = \frac{1}{2}, \gamma = \frac{\pi}{4}$

$$\Rightarrow \tan(\alpha + \beta) = \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta} = \frac{\frac{1}{3} + \frac{1}{2}}{1 - \frac{1}{3} \times \frac{1}{2}} = \frac{\frac{5}{6}}{1 - \frac{1}{6}} = 1$$

$\Rightarrow \alpha + \beta = \frac{\pi}{4}$, 又 $\gamma = \frac{\pi}{4} \therefore \alpha + \beta + \gamma = \frac{\pi}{4} + \frac{\pi}{4} = \frac{\pi}{2} = 90^\circ$ 即為所求

(2) 由題圖可得 $\angle FAH = \beta - \alpha$

$$\begin{aligned} \Rightarrow \tan \angle FAH = \tan(\beta - \alpha) &= \frac{\tan \beta - \tan \alpha}{1 + \tan \beta \tan \alpha} = \frac{\frac{1}{2} - \frac{1}{3}}{1 + \frac{1}{2} \times \frac{1}{3}} = \frac{\frac{1}{6}}{1 + \frac{1}{6}} \\ &= \frac{1}{7} \text{ 即為所求} \end{aligned}$$