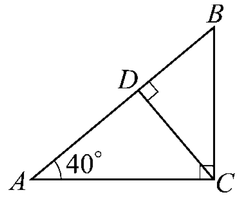
高雄市明誠中學 高一數學平時測驗 日期：99.04.27					
範圍	2-1 銳角三角函數	班級		姓名	
		座號			

一、單選題 (每題 5 分)

( ) 1.$\triangle ABC$ 中，$\angle C = 90°$，$\angle A = 40°$，$\overline{CD}$ 垂直 $\overline{AB}$ 於 D，已知 $\overline{AB} = \alpha$，求 $\overline{CD} =$ (1) $\alpha \sin 40° \cos 40°$ (2) $\alpha \tan 40° \sin 40°$ (3) $\alpha \tan 40° \cos 40°$ (4) $\alpha \sin^2 40°$ (5) $\alpha \cos^2 40°$.

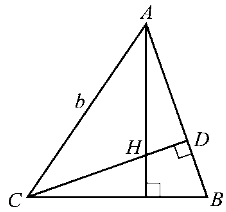


解答 1

解析 $\triangle ABC$ 中，$\sin 40° = \dfrac{\overline{BC}}{\overline{AB}} \Rightarrow \overline{BC} = \alpha \cdot \sin 40°$

$\triangle BCD$ 中，$\sin 50° = \dfrac{\overline{CD}}{\overline{BC}} \Rightarrow \overline{CD} = \overline{BC} \cdot \sin 50° = \alpha \sin 40° \sin 50° = \alpha \sin 40° \cos 40°$

( ) 2.設 H 爲 $\triangle ABC$ 三高的交點，若以 b 表 $\overline{CA}$ 的長度，則線段 AH 的長度等於(1) $b\cos A \sec B$ (2) $b\cos A \cos B$ (3) $b \cos A \csc B$ (4) $b \cos A \tan B$.



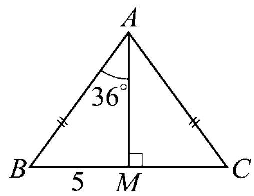
解答 3

解析 $\triangle ACD$ 中，$\cos A = \dfrac{\overline{AD}}{b} \Rightarrow \overline{AD} = b \cdot \cos A$

$\triangle AHD$ 中，$\angle AHD = \angle B$，$\csc(\angle AHD) = \dfrac{\overline{AH}}{\overline{AD}}$，$\csc B = \dfrac{\overline{AH}}{b \cdot \cos A} \Rightarrow \overline{AH} = b \cos A \csc B$

( ) 3.有一等腰三角形，底邊爲 10，頂角 $72°$，下列何者可以表示腰長?

(1) $5\sin 36°$ (2) $5\tan 36°$ (3) $5\cot 36°$ (4) $5\sec 36°$ (5) $5\csc 36°$.



解答 5

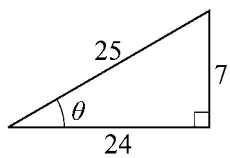
解析 $\triangle ABC$ 中，$\overline{AM}$ 爲中線，$\angle BAM = 36°$

$\csc 36° = \dfrac{\overline{AB}}{5} \Rightarrow \overline{AB} = 5\csc 36°$.

( ) 4.設 θ 爲銳角，若 $\cos\theta = \dfrac{24}{25}$，則下列何者爲真?

(1) $\sec\theta = \dfrac{7}{25}$ (2) $\tan\theta = \dfrac{24}{7}$ (3) $\csc\theta = \dfrac{25}{24}$ (4) $\cot\theta = \dfrac{24}{7}$ (5)以上皆非 .

解答 4



解析 $\cos\theta = \dfrac{24}{25} \Rightarrow \cot\theta = \dfrac{24}{7}$，$\sec\theta = \dfrac{25}{24}$，$\tan\theta = \dfrac{7}{24}$，$\csc\theta = \dfrac{25}{7}$

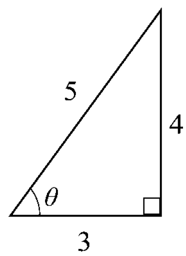
二、填充題 (每題 10 分)

1. θ 爲銳角，若 $\tan\theta = \dfrac{4}{3}$，求 $\dfrac{\cos\theta + \sin\theta \cdot \sec\theta}{\sec^2\theta} =$ ____________ .

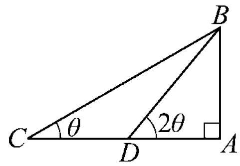
解答 $\dfrac{87}{125}$

解析 $\because \tan\theta=\dfrac{4}{3}$，如圖

$\sin\theta=\dfrac{4}{5}$，$\cos\theta=\dfrac{3}{5}$，$\sec\theta=\dfrac{5}{3}$，則 $\dfrac{\cos\theta+\sin\theta\cdot\sec\theta}{\sec^2\theta}=\dfrac{\frac{3}{5}+\frac{4}{5}\cdot\frac{5}{3}}{\left(\frac{5}{3}\right)^2}=\dfrac{87}{125}$．



2. 如下圖，直角△ ABC 中，$\angle A=90^\circ$，$\angle ADB=2\theta$，$\tan\theta=\dfrac{1}{2}$，求 $\tan 2\theta=$____________．

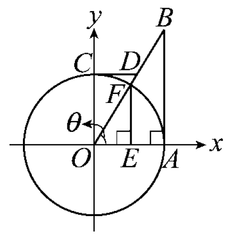


解答 $\dfrac{4}{3}$

解析 △ ABC 中，$\tan\theta=\dfrac{1}{2}$，設 $\overline{AB}=1$，$\overline{AC}=2$，$\overline{CD}=x=\overline{BD}$，$\overline{AD}=2-x$

△ ABD 中，$1^2+(2-x)^2=x^2\Rightarrow x=\dfrac{5}{4}$，$\tan 2\theta=\dfrac{\overline{AB}}{\overline{AD}}=\dfrac{1}{2-\frac{5}{4}}=\dfrac{4}{3}$．

3. 如圖，$0^\circ<\theta<90^\circ$，$\overline{AB}$與$\overline{CD}$爲單位圓（半徑爲 1 的圓）的切線段，試以 $\sin\theta$，$\cos\theta$，$\tan\theta$，$\cot\theta$，$\sec\theta$，$\csc\theta$ 表示下列線段長:
(1) $\overline{AB}=$____________，(2) $\overline{EF}=$____________，(3) $\overline{OE}=$____________，
(4) $\overline{OB}=$____________，(5) $\overline{CD}=$____________，(6) $\overline{OD}=$____________．



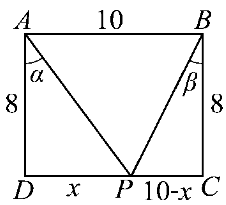
解答 (1) $\tan\theta$；(2) $\sin\theta$；(3) $\cos\theta$；(4) $\sec\theta$；(5) $\cot\theta$；(6) $\csc\theta$

解析 △ OAB 中，$\tan\theta=\dfrac{\overline{AB}}{\overline{OA}}=\dfrac{\overline{AB}}{1}\Rightarrow\overline{AB}=\tan\theta$，　$\sec\theta=\dfrac{\overline{OB}}{\overline{OA}}=\dfrac{\overline{OB}}{1}\Rightarrow\overline{OB}=\sec\theta$

△ OEF 中，$\sin\theta=\dfrac{\overline{EF}}{\overline{OF}}=\dfrac{\overline{EF}}{1}\Rightarrow\overline{EF}=\sin\theta$，　$\cos\theta=\dfrac{\overline{OE}}{\overline{OF}}=\dfrac{\overline{OE}}{1}\Rightarrow\overline{OE}=\cos\theta$

△ OCD 中，$\csc\theta=\dfrac{\overline{OD}}{\overline{OC}}=\dfrac{\overline{OD}}{1}\Rightarrow\overline{OD}=\csc\theta$，　$\cot\theta=\dfrac{\overline{CD}}{\overline{OC}}=\dfrac{\overline{CD}}{1}\Rightarrow\overline{CD}=\cot\theta$．

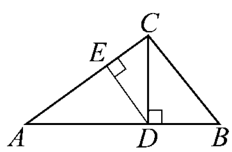
4. 矩形 $ABCD$ 中，已知 $\overline{AB}=10$，$\overline{BC}=8$，若點 P 爲 $\overline{CD}$ 邊上一點，設 $\angle PAD=\alpha$，$\angle PBC=\beta$，則 $\tan\alpha+\tan\beta=$____________．



解答 $\dfrac{5}{4}$

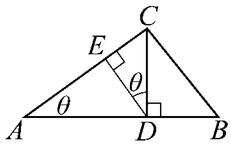
解析 設 $\overline{PD}=x$，則 $\overline{PC}=10-x$，$\tan\alpha+\tan\beta=\dfrac{x}{8}+\dfrac{10-x}{8}=\dfrac{10}{8}=\dfrac{5}{4}$．

5. 直角△ ABC 中，$\angle C=90^\circ$，$\overline{AC}=4$，$\overline{BC}=3$，自 C 作 $\overline{CD}$ 垂直 $\overline{AB}$ 於 D，作 $\overline{DE}$ 垂直 $\overline{AC}$ 於 E，求 $\overline{DE}=$____________．



解答 $\dfrac{48}{25}$

解析　令$\angle A=\theta$，則$\angle CDE=\theta$，



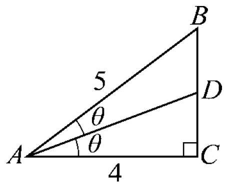
$\triangle ABC$中，$\sin\theta=\dfrac{\overline{BC}}{\overline{AB}}=\dfrac{3}{5}$，$\cos\theta=\dfrac{\overline{AC}}{\overline{AB}}=\dfrac{4}{5}$

$\triangle ACD$中，$\sin\theta=\dfrac{\overline{CD}}{\overline{AC}}\Rightarrow\overline{CD}=\overline{AC}\times\sin\theta=4\times\dfrac{3}{5}=\dfrac{12}{5}$

$\triangle CDE$中，$\cos\theta=\dfrac{\overline{DE}}{\overline{CD}}\Rightarrow\overline{DE}=\overline{CD}\times\cos\theta=\dfrac{12}{5}\times\dfrac{4}{5}=\dfrac{48}{25}$．

6. 直角$\triangle ABC$中，$\angle C=90^\circ$，若$\overline{AB}=5$，$\overline{AC}=4$，$\angle A$的平分線交$\overline{BC}$於D，$\angle DAB=\theta$，求$\tan\theta=$____________．

解答　$\dfrac{1}{3}$



解析　$\overline{AB}=5$，$\overline{AC}=4\Rightarrow\overline{BC}=3$，由分角線性質$\dfrac{\overline{BD}}{\overline{DC}}=\dfrac{\overline{AB}}{\overline{AC}}=\dfrac{5}{4}$，

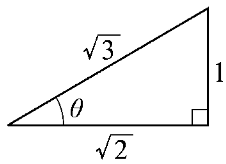
$\therefore\overline{BD}=\dfrac{5}{9}\times3=\dfrac{5}{3}$，$\overline{CD}=\dfrac{4}{9}\times3=\dfrac{4}{3}$，$\triangle ACD$中，$\tan\theta=\dfrac{\overline{CD}}{\overline{AC}}=\dfrac{\frac{4}{3}}{4}=\dfrac{1}{3}$．

7. 設θ為一銳角，且$\dfrac{1-\sin\theta}{1+\sin\theta}=2-\sqrt{3}$，試求$\tan\theta+\cot\theta=$____________．

解答　$\dfrac{3\sqrt{2}}{2}$

解析　$\dfrac{1-\sin\theta}{1+\sin\theta}=2-\sqrt{3}\Rightarrow1-\sin\theta=\left(2-\sqrt{3}\right)\left(1+\sin\theta\right)\Rightarrow\left(3-\sqrt{3}\right)\sin\theta=\sqrt{3}-1$

$\Rightarrow\sin\theta=\dfrac{\sqrt{3}-1}{3-\sqrt{3}}=\dfrac{1}{\sqrt{3}}$，得$\tan\theta=\dfrac{1}{\sqrt{2}}$，$\cot\theta=\sqrt{2}$



$\therefore\tan\theta+\cot\theta=\dfrac{1}{\sqrt{2}}+\sqrt{2}=\dfrac{3\sqrt{2}}{2}$．

8. $\sin30^\circ-\tan45^\circ\cdot\cos^2 30^\circ+\sqrt{3}\sec30^\circ-\csc60^\circ=a+b\sqrt{3}$（$a$，$b$為有理數），求數對$\left(a,b\right)=$__________．

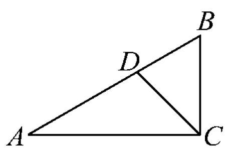
解答　$\left(\dfrac{7}{4},-\dfrac{2}{3}\right)$

解析　原式$=\dfrac{1}{2}-1\times(\dfrac{\sqrt{3}}{2})^2+\sqrt{3}\times\dfrac{2}{\sqrt{3}}-\dfrac{2}{\sqrt{3}}$

$=\dfrac{1}{2}-\dfrac{3}{4}+2-\dfrac{2}{\sqrt{3}}=\dfrac{1}{2}-\dfrac{3}{4}+2-\dfrac{2\sqrt{3}}{3}=\dfrac{7}{4}-\dfrac{2}{3}\sqrt{3}=a+b\sqrt{3}$

$a=\dfrac{7}{4}$，$b=-\dfrac{2}{3}$，$\therefore\left(a,b\right)=\left(\dfrac{7}{4},-\dfrac{2}{3}\right)$．

9. 如圖，$\overline{CD}\perp\overline{AB}$，$\overline{AC}\perp\overline{BC}$，$\overline{AC}=8$，$\overline{BC}=6$，$\angle BCD=\theta$，則$\sin\theta=$__________．



解答 $\frac{3}{5}$

解析 $\because \theta+\angle ACD=\angle ACD+\angle CAD=90°$ ，$\therefore \angle CAD=\theta$ $\therefore \sin\theta=\frac{\overline{BC}}{\overline{AB}}=\frac{6}{10}=\frac{3}{5}$.

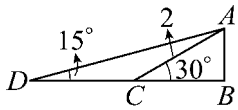
10.直角△ABC 中，$\angle ACB=30°$，$\overline{AB}=1$，$\overline{BC}=\sqrt{3}$，在 $\overline{BC}$ 上取一點 D，使 $\overline{CD}=\overline{CA}$，求

(1) $\sin 15°=$____________，(2) $\cos 15°=$____________，(3) $\tan 15°=$____________ .

解答 (1) $\frac{\sqrt{6}-\sqrt{2}}{4}$;(2) $\frac{\sqrt{6}+\sqrt{2}}{4}$;(3) $2-\sqrt{3}$

解析 $\overline{AD}^2=\overline{AB}^2+\overline{BD}^2=1^2+\left(2+\sqrt{3}\right)^2=8+2\sqrt{12}\Rightarrow \overline{AD}=\sqrt{8+2\sqrt{12}}=\sqrt{6}+\sqrt{2}$

$$\sin 15°=\frac{\overline{AB}}{\overline{AD}}=\frac{1}{\sqrt{6}+\sqrt{2}}=\frac{\sqrt{6}-\sqrt{2}}{4}$$



$$\cos 15°=\frac{\overline{BD}}{\overline{AD}}=\frac{2+\sqrt{3}}{\sqrt{6}+\sqrt{2}}=\frac{2+\sqrt{3}}{\sqrt{6}+\sqrt{2}}\times\frac{\sqrt{6}-\sqrt{2}}{\sqrt{6}-\sqrt{2}}=\frac{\sqrt{6}+\sqrt{2}}{4}$$

$$\tan 15°=\frac{\overline{AB}}{\overline{BD}}=\frac{1}{2+\sqrt{3}}=2-\sqrt{3}\ .$$

11.求下列各式的值:

(1) $\sqrt{3}\tan 30°+\sqrt{2}\sin 45°+\cos 60°=$__________ . (2) $\frac{4}{3}\sin^2 60°-\frac{1}{2}\tan^2 45°-\frac{2}{3}\cos^2 30°=$___________ .

解答 (1) $\frac{5}{2}$;(2) 0

解析 (1)原式 $=\sqrt{3}\times\frac{1}{\sqrt{3}}+\sqrt{2}\times\frac{\sqrt{2}}{2}+\frac{1}{2}=\frac{5}{2}$.

(2)原式 $=\frac{4}{3}\times\left(\frac{\sqrt{3}}{2}\right)^2-\frac{1}{2}\times 1^2-\frac{2}{3}\times\left(\frac{\sqrt{3}}{2}\right)^2=1-\frac{1}{2}-\frac{1}{2}=0$.

12.求 $\sqrt{3}\tan 30°+\sqrt{2}\sin 45°+\cos 60°=$____________ .

解答 $\frac{5}{2}$

解析 原式 $=\sqrt{3}\cdot\frac{1}{\sqrt{3}}+\sqrt{2}\cdot\frac{\sqrt{2}}{2}+\frac{1}{2}=1+1+\frac{1}{2}=\frac{5}{2}$.

13.設 A，B，C，D 均爲銳角，若 $\sin A=\frac{\sqrt{2}}{2}$，$\cos B=\frac{1}{2}$，$\tan C=\sqrt{3}$，$\sec D=2$，求 $\angle A+\angle B+\angle C+\angle D=$

__________ .

解答 225°

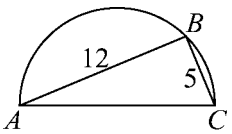
解析 $\sin A=\frac{\sqrt{2}}{2}\Rightarrow\angle A=45°$；$\cos B=\frac{1}{2}\Rightarrow\angle B=60°$；$\tan C=\sqrt{3}\Rightarrow C=60°$；$\sec D=2$，$\angle D=60°$

14. $\frac{1+\cos 30°-\tan 45°}{1+\csc 60°-\cot 45°}=$____________ .

解答 $\frac{3}{4}$

解析 原式$=\dfrac{1+\frac{\sqrt{3}}{2}-1}{1+\frac{2}{\sqrt{3}}-1}=\dfrac{\frac{\sqrt{3}}{2}}{\frac{2}{\sqrt{3}}}=\dfrac{3}{4}$.

15.如圖，已知$\overline{AC}$爲半圓的直徑，B爲半圓上一點，$\overline{AB}=12$，$\overline{BC}=5$，則 $\cos A=$____________ .



解答 $\frac{12}{13}$

解析 半圓內的圓周角爲直角$\therefore \angle B=90^\circ$，$\overline{AC}=\sqrt{12^2+5^2}=13$，$\cos A=\dfrac{\overline{AB}}{\overline{AC}}=\dfrac{12}{13}$.

16.設θ爲銳角，滿足$2\cos^2\theta+3\cos\theta-2=0$，求$\theta=$____________ .

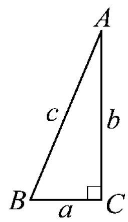
解答 60°

解析 $2\cos^2\theta+3\cos\theta-2=0\Rightarrow(2\cos\theta-1)(\cos\theta+2)=0$

$\Rightarrow \cos\theta=\frac{1}{2}$ 或 $\cos\theta=-2$（不合），$\therefore \theta=60^\circ$.

17.△ABC中，$\angle C=90^\circ$，若$\cos A+8\cos B=4$，求$a:b:c=$____________ .

解答 5:12:13



解析 $\cos A+8\cos B=4\Rightarrow\frac{b}{c}+8\cdot\frac{a}{c}=4\Rightarrow b=4c-8a\ldots(1)$

又 $a^2+b^2=c^2\ldots(2)$

(1)代入(2) $a^2+(4c-8a)^2=c^2\Rightarrow 65a^2-64ac+15c^2=0\Rightarrow 65(\frac{a}{c})^2-64(\frac{a}{c})+15=0$

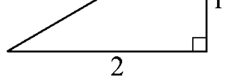
設$t=\frac{a}{c}\Rightarrow 65t^2-64t+15=0$,$(13t-5)(5t-3)=0$, $t=\frac{5}{13},\frac{3}{5}$

$\Rightarrow \frac{a}{c}=\frac{5}{13}$ 或 $\frac{a}{c}=\frac{3}{5}$（代入(1)不合），$c=\frac{13}{5}a, b=\frac{52}{5}a-8a=\frac{12}{5}a$，

$\therefore a:b:c=a:\frac{12}{5}a:\frac{13}{5}a=5:12:13$.

18.設θ是一銳角，已知$2\sin\theta=\cos\theta$，求$\csc\theta=$____________ .

解答 $\sqrt{5}$



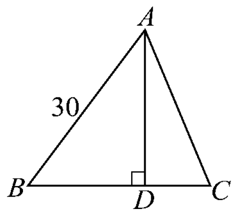
解析 $2\sin\theta=\cos\theta\Rightarrow\frac{\sin\theta}{\cos\theta}=\frac{1}{2}\Rightarrow\tan\theta=\frac{1}{2}$ $\therefore \csc\theta=\frac{\sqrt{5}}{1}=\sqrt{5}$.

19.△ABC中，已知$\overline{AB}=30$，$\sin B=\frac{4}{5}$，$\cos C=\frac{5}{13}$，$\overline{AD}\perp\overline{BC}$，求

(1) $\overline{AD}=$______，(2) $\overline{AC}=$________ .

解答 (1)24;(2)26

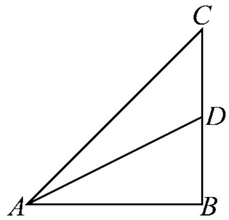
解析 $\triangle ABD$ 中，$\sin B=\dfrac{\overline{AD}}{\overline{AB}}\Rightarrow\dfrac{4}{5}=\dfrac{\overline{AD}}{30}\Rightarrow\overline{AD}=24$



$\triangle ACD$ 中，$\cos C=\dfrac{5}{13}\Rightarrow\sin C=\dfrac{12}{13}$

又 $\sin C=\dfrac{\overline{AD}}{\overline{AC}}\Rightarrow\dfrac{12}{13}=\dfrac{24}{\overline{AC}}\Rightarrow\overline{AC}=26$.

20.$\triangle ABC$ 是等腰直角三角形，若 $\angle B=90^\circ$ 且 D 是 $\overline{BC}$ 邊的中點，則
(1) $\tan\angle BAD=$____________，(2) $\tan\angle DAC=$____________ .

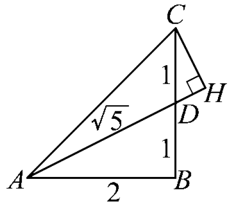


解答 (1)$\dfrac{1}{2}$;(2)$\dfrac{1}{3}$

解析 令 $\overline{AB}=\overline{BC}=2$，則 $\overline{DB}=1$，$\overline{AC}=2\sqrt{2}$，$\tan\angle BAD=\dfrac{1}{2}$

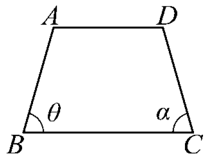
延長 $\overline{AD}$，作 $\overline{CH}\perp\overline{AD}$ 交 $\overline{AD}$ 於 H，$\triangle CHD\sim\triangle ABD$

$\dfrac{\overline{DH}}{\overline{BD}}=\dfrac{\overline{CD}}{\overline{AD}}=\dfrac{\overline{CH}}{\overline{AB}}\Rightarrow\dfrac{\overline{DH}}{1}=\dfrac{1}{\sqrt{5}}=\dfrac{\overline{CH}}{2}\Rightarrow\overline{DH}=\dfrac{1}{\sqrt{5}}$，$\overline{CH}=\dfrac{2}{\sqrt{5}}$



$\tan\angle DAC=\dfrac{\overline{CH}}{\overline{AH}}=\dfrac{\frac{2}{\sqrt{5}}}{\sqrt{5}+\frac{1}{\sqrt{5}}}=\dfrac{2}{5+1}=\dfrac{1}{3}$.

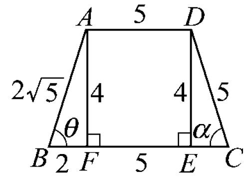
21.如下圖梯形 $ABCD$ 中，$\overline{AD}=\overline{CD}=5$，$\overline{BC}=10$，$\theta$ 爲銳角，且 $\sin\alpha=\dfrac{4}{5}$，求 $\cos\theta=$____________ .



解答 $\dfrac{\sqrt{5}}{5}$

解析 $\sin\alpha=\dfrac{4}{5}\Rightarrow\overline{DE}=4=\overline{AF}$，$\overline{CE}=3$，$\therefore\overline{BF}=10-5-3=2$，

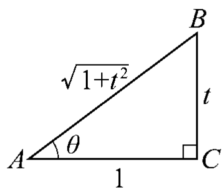
$\overline{AB}=\sqrt{2^2+4^2}=2\sqrt{5}$，故 $\cos\theta=\dfrac{2}{2\sqrt{5}}=\dfrac{1}{\sqrt{5}}=\dfrac{\sqrt{5}}{5}$.



22.設 θ 爲銳角，若 $\tan\theta=t$，試以 t 表出 $\sin\theta=$____________ .

解答 $\dfrac{t}{\sqrt{1+t^2}}$

解析 令 $\tan\theta=\dfrac{t}{1}$，則 $\overline{AB}=\sqrt{1+t^2}$，



$\sin\theta=\dfrac{\overline{BC}}{\overline{AB}}=\dfrac{t}{\sqrt{1+t^2}}=\dfrac{\tan\theta}{\sqrt{1+\tan^2\theta}}$.