

高雄市明誠中學 高一數學平時測驗 日期：99.03.02				
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一、單選題 (每題 5 分)

() 1. 下列何者為真?

(1) $(-27)^{\frac{1}{3}} = -3$ (2) $8^{\frac{1}{3}} = -2$ (3) $125^{\frac{4}{3}} = 625$ (4) $\sqrt[3]{-8} = \sqrt[6]{(-8)^2}$

(5) $\sqrt{7-4\sqrt{3}} = \sqrt{3}-2$.

解答 3

解析 (1) $(-27)^{\frac{1}{3}} \Rightarrow -27 < 0$ 無意義 .

(2) $8^{\frac{1}{3}} = (2^3)^{\frac{1}{3}} = 2^{-1} = \frac{1}{2}$.

(3) $125^{\frac{4}{3}} = (5^3)^{\frac{4}{3}} = 5^4 = 625$.

(4) $\sqrt[3]{-8} = [(-2)^3]^{\frac{1}{3}} = -2$ $\sqrt[6]{(-8)^2} = \sqrt[6]{64} = (2^6)^{\frac{1}{6}} = 2$.

(5) $\sqrt{7-4\sqrt{3}} = \sqrt{7-2\sqrt{12}} = \sqrt{4-\sqrt{3}} = 2-\sqrt{3}$.

二、多選題 (每題 10 分)

() 1. 下列各選項何者為真? (1) $2^3 = 6$ (2) $2^0 = 0$ (3) $2^{-1} = \frac{1}{2}$ (4) $2^{\frac{1}{2}} = \sqrt{2}$.

解答 34

解析 (1) $2^3 = 2 \cdot 2 \cdot 2 = 8$.

(2) $2^0 = 1$.

(3) $2^{-1} = \frac{1}{2^1} = \frac{1}{2}$.

(4) $2^{\frac{1}{2}} = \sqrt{2^1} = \sqrt{2}$.

三、填充題 (每題 10 分)

1. 化簡 $(\sqrt{3})^{-2} \times [(\sqrt{3})^3]^{-\frac{1}{2}} \times (\sqrt{3})^{\frac{11}{2}} = \underline{\hspace{2cm}}$.

解答 3

解析 原式 = $\left(3^{\frac{1}{2}}\right)^{-2} \times \left[\left(3^{\frac{1}{2}}\right)^3\right]^{-\frac{1}{2}} \times \left(3^{\frac{1}{2}}\right)^{\frac{11}{2}} = (3^{-1}) \times \left(3^{-\frac{3}{4}}\right) \times \left(3^{\frac{11}{4}}\right) = 3^{-1 + \left(-\frac{3}{4}\right) + \frac{11}{4}} = 3^1 = 3$.

2. 化簡 $\sqrt[5]{7^{10}} \times \sqrt[3]{\sqrt{7^{12}}} - 7^3 =$ _____ .

解答 2058

解析 原式 $\Rightarrow (7^{10})^{\frac{1}{5}} \times \left[(7^{12})^{\frac{1}{2}} \right]^{\frac{1}{3}} - 7^3 = 7^2 \times 7^2 - 7^3 = 49 \times 49 - 343 = 2058$.

3. 化簡 $5^{2.3} \times 5^{-0.8} \div 5^{2.5} - 5^{-2} =$ _____ .

解答 $\frac{4}{25}$

解析 原式 $= 5^{2.3+(-0.8)-2.5} - \frac{1}{25} = 5^{-1} - \frac{1}{25} = \frac{1}{5} - \frac{1}{25} = \frac{4}{25}$.

4. 求 $8^{\frac{2}{3}} + 25^{\frac{3}{2}} + 27^{\frac{1}{3}} =$ _____ .

解答 132

解析 $8^{\frac{2}{3}} + 25^{\frac{3}{2}} + 27^{\frac{1}{3}} = (2^3)^{\frac{2}{3}} + (5^2)^{\frac{3}{2}} + (3^3)^{\frac{1}{3}} = 2^2 + 5^3 + 3^1 = 4 + 125 + 3 = 132$.

5. 求 $(4.3^2 - 3.4^2)^0 =$ _____ .

解答 1

解析 $4.3^2 - 3.4^2 \neq 0 \quad \therefore (4.3^2 - 3.4^2)^0 = 1$.

6. 化簡 $\sqrt[4]{a^{20}} \times \sqrt[3]{\sqrt{a^{12}}} =$ _____ .

解答 a^7

解析 原式 $= (a^{20})^{\frac{1}{4}} \times \left(a^{\frac{12}{2}} \right)^{\frac{1}{3}} = a^5 \times a^2 = a^7$.

7. 已知 $(\sqrt[3]{2})^x \times \sqrt{(\sqrt{2})^3} = 2$, 求 $x =$ _____ .

解答 $\frac{3}{4}$

解析 原式 $\Rightarrow \left(\frac{1}{2^3} \right)^x \times \left[\left(\frac{1}{2^2} \right)^3 \right]^{\frac{1}{2}} = 2 \Rightarrow 2^{\frac{x}{3}} \times 2^{\frac{3}{4}} = 2 \Rightarrow 2^{\frac{x}{3} + \frac{3}{4}} = 2 \Rightarrow \frac{x}{3} + \frac{3}{4} = 1 \Rightarrow x = \frac{3}{4}$.

8. 化簡 $(2 + \sqrt{3})^{\frac{4}{3}} \times (2 - \sqrt{3})^{\frac{4}{3}} =$ _____ .

解答 1

解析 原式 $\Rightarrow \left[(2 + \sqrt{3})(2 - \sqrt{3}) \right]^{\frac{4}{3}} = 1^{\frac{4}{3}} = 1$.

9. 設 $a^{2x} = 3$, 求

$$(1) \frac{a^{3x} + a^{-3x}}{a^x + a^{-x}} = \underline{\hspace{2cm}} \quad (2) \frac{a^{3x} + a^{-3x}}{a^x - a^{-x}} = \underline{\hspace{2cm}} .$$

解答 (1) $\frac{7}{3}$; (2) $\frac{14}{3}$

解析 (1) $\frac{a^{3x} + a^{-3x}}{a^x + a^{-x}} = \frac{(a^x + a^{-x})(a^{2x} - 1 + a^{-2x})}{a^x + a^{-x}} = a^{2x} - 1 + a^{-2x} = 3 - 1 + \frac{1}{3} = \frac{7}{3} .$

$$(2) \frac{a^{3x} + a^{-3x}}{a^x - a^{-x}} = \frac{a^{4x} + a^{-2x}}{a^{2x} - 1} \quad (\text{分子、分母同乘 } a^x) = \frac{9 + \frac{1}{3}}{3 - 1} = \frac{\frac{28}{3}}{2} = \frac{14}{3} .$$

10. 設 $a^x + a^{-x} = 5$, 求 $a^{3x} + a^{-3x} = \underline{\hspace{2cm}} .$

解答 110

解析 $a^{3x} + a^{-3x} = (a^x)^3 + (a^{-x})^3 = (a^x + a^{-x})^3 - 3a^x \cdot a^{-x}(a^x + a^{-x}) = 5^3 - 3 \cdot 1 \cdot 5 = 110 .$

11. 設 $a^{\frac{1}{2}} - a^{-\frac{1}{2}} = 2\sqrt{3}$, 求下列各式的值

$$(1) a + a^{-1} = \underline{\hspace{2cm}} \quad (2) a^{\frac{1}{2}} + a^{-\frac{1}{2}} = \underline{\hspace{2cm}} \quad (3) a^{\frac{3}{2}} + a^{-\frac{3}{2}} = \underline{\hspace{2cm}} .$$

解答 (1) 14; (2) 4; (3) 52

解析 (1) $\because a^{\frac{1}{2}} - a^{-\frac{1}{2}} = 2\sqrt{3} \quad \Rightarrow \left(a^{\frac{1}{2}} - a^{-\frac{1}{2}}\right)^2 = (2\sqrt{3})^2 \Rightarrow a + a^{-1} - 2 = 12 \Rightarrow a + a^{-1} = 14 .$

$$(2) \left(a^{\frac{1}{2}} + a^{-\frac{1}{2}}\right)^2 = a + a^{-1} + 2 = 14 + 2 = 16 \quad \therefore a^{\frac{1}{2}} + a^{-\frac{1}{2}} = \pm 4 \quad (\text{取正}) .$$

$$(3) a^{\frac{3}{2}} + a^{-\frac{3}{2}} = \left(a^{\frac{1}{2}}\right)^3 + \left(a^{-\frac{1}{2}}\right)^3 = \left(a^{\frac{1}{2}} + a^{-\frac{1}{2}}\right)^3 - 3 \cdot a^{\frac{1}{2}} \cdot a^{-\frac{1}{2}} \left(a^{\frac{1}{2}} + a^{-\frac{1}{2}}\right) = 4^3 - 3 \cdot 1 \cdot 4 = 52 .$$

12. 設 $(2.5)^x = (0.25)^y = 1000$, 求 $\frac{1}{x} - \frac{1}{y} = \underline{\hspace{2cm}} .$

解答 $\frac{1}{3}$

解析 $(2.5)^x = 1000 \Rightarrow 2.5 = 1000^{\frac{1}{x}} \dots (1)$

$$(0.25)^y = 1000 \Rightarrow 0.25 = 1000^{\frac{1}{y}} \dots (2)$$

$$\begin{aligned} (1) \text{ 得 } \quad \frac{2.5}{0.25} &= 1000^{\frac{1}{x} - \frac{1}{y}} \Rightarrow 10 = 10^{3\left(\frac{1}{x} - \frac{1}{y}\right)} \quad \therefore \frac{1}{x} - \frac{1}{y} = \frac{1}{3} . \\ (2) \end{aligned}$$

13. 設 $67^x = 27$, $603^y = 81$, 求 $\frac{3}{x} - \frac{4}{y} =$ _____ .

解答 -2

解析 $67^x = 27 \Rightarrow 67 = 27^{\frac{1}{x}} = 3^{\frac{3}{x}} \dots (1)$

$603^y = 81 \Rightarrow 603 = 81^{\frac{1}{y}} = 3^{\frac{4}{y}} \dots (2)$

(1) 得 $\frac{1}{9} = 3^{\frac{3-4}{x \cdot y}} \quad \therefore \frac{3}{x} - \frac{4}{y} = -2$.

14. 設 $\sqrt[3]{27^{x-1}} = \sqrt[3]{9}$ 且 $2^x = \left(\frac{1}{8}\right)^{-y}$, 求 $x - y =$ _____ .

解答 2

解析 $\sqrt[3]{27^{x-1}} = \sqrt[3]{9} \Rightarrow 3^{\frac{3x-3}{x}} = 3^{\frac{2}{x}}$

$3 - \frac{3}{x} = \frac{2}{x} \Rightarrow \frac{3}{x} + \frac{2}{y} = 3 \dots (1)$

$2^x = \left(\frac{1}{8}\right)^{-y} \Rightarrow 2^x = 2^{3y} \Rightarrow x = 3y \dots (2)$

解(1)(2)得 $x = 3, y = 1 \quad \therefore x - y = 2$.

15. 解方程式 $(\sqrt{3})^{3x} \cdot 3^x = \frac{27\sqrt{3}}{3^x}$ 得 $x =$ _____ .

解答 1

解析 原式 $\Rightarrow 3^{\frac{3x}{2}} \cdot 3^x = \frac{3^3 \cdot 3^{\frac{1}{2}}}{3^x} \Rightarrow 3^{\frac{5x}{2}} = 3^{\frac{7}{2-x}} \Rightarrow \frac{5}{2}x = \frac{7}{2-x} \Rightarrow \frac{7}{2}x = \frac{7}{2} \Rightarrow x = 1$.

16. 依據實驗, 某種細菌原有的數目為 N , 經過 x 天後細菌的數目變成 $N \cdot a^x$, 已知 2 天後, 5 天後細菌的數目依序為 $3 \cdot 10^6$, $(2.4) \cdot 10^7$, 則(1) $a =$ _____ (2) 8 天後細菌的數目為 _____ .

解答 (1) 2; (2) 1.92×10^8

解析 (1) 由題意 $Na^2 = 3 \cdot 10^6 \dots (1)$

$Na^5 = (2.4) \cdot 10^7 \dots (2)$

(2) $a^3 = \frac{(2.4) \cdot 10^7}{3 \cdot 10^6} = 8 \Rightarrow a = 2$.

(2) 8 天後 $Na^8 = Na^5 \cdot a^3 = (2.4) \cdot 10^7 \cdot 2^3 = 1.92 \times 10^8$.

17. 已知 x 為實數, 若 $x^{x+4} = x^7$, 求 $x =$ _____ .

解答 0, 1, -1, 3

解析 ① $x=0$, 左 $=0^4=0$, 右 $=0^7=0$, 成立

② $x=1$, 左 $=1^5=1$, 右 $=1^7=1$, 成立

③ $x=-1$, 左 $=(-1)^3=-1$, 右 $=(-1)^7=-1$, 成立

④ $x \neq 0, 1, -1$, 則 $x+4=7 \Rightarrow x=3$

$\therefore x=0, 1, -1, 3$.

18.完成下列的填空：

(1) $2^3 \cdot 2^4 = 2^{\square}$. (2) $(-3)^3 \cdot (-3)^5 = 9^{\square}$. (3) $(4^{\square})^2 = 2^{20}$.

(4) $3^5 \cdot 4^5 = \square^5$. (5) $(-2)^{10} \cdot 3^5 = \square^5$.

解答 (1)7;(2)4;(3)5;(4)12;(5)12

解析 (1) $2^3 \cdot 2^4 = 2^{3+4} = 2^7$.

(2) $(-3)^3 \cdot (-3)^5 = (-3)^{3+5} = (-3)^8 = 3^8 = 9^4$.

(3) $2^{20} = (2^{10})^2 = (4^5)^2$.

(4) $3^5 \cdot 4^5 = (3 \cdot 4)^5 = 12^5$.

(5) $(-2)^{10} \cdot 3^5 = ((-2)^2)^5 \cdot 3^5 = 4^5 \cdot 3^5 = 12^5$.

19.已知 $2^{0.03} = x$, $4^{0.4} = y$, 若 $2^{-1.17} = kxy$, 求 $k =$ _____ .

解答 $\frac{1}{4}$

解析 $\because 2^{-1.17} = kxy = k \cdot 2^{0.03} \cdot 4^{0.4} = k \cdot 2^{0.03} \cdot 2^{0.8} = k \cdot 2^{0.83}$

$\therefore k = \frac{2^{-1.17}}{2^{0.83}} = 2^{-1.17-0.83} = 2^{-2} = \frac{1}{4}$.