高雄市明誠中學 高一數學平時測驗					日期：96.07.16
範圍	3-5 複數極式	班級	普一　　班	姓名	
		座號			

一、填充題(每題 10 分)

1. 若$z=\sqrt{2}-\sqrt{6}i=r(\cos\theta+i\sin\theta)$，$r>0$，$0\le\theta<2\pi$，則數對$(r，\theta)=$ __________ 。

【解答】$(2\sqrt{2},\frac{5\pi}{3})$

【詳解】

$$z=\sqrt{2}-\sqrt{6}i=2\sqrt{2}(\frac{1}{2}-\frac{\sqrt{3}}{2}i)=2\sqrt{2}(\cos\frac{5\pi}{3}+i\sin\frac{5\pi}{3})$$ ，故數對$(r，\theta)=(2\sqrt{2},\frac{5\pi}{3})$

2. $\dfrac{(\cos 221^\circ+i\sin 139^\circ)(\cos 39^\circ-i\sin 39^\circ)}{\sin 350^\circ+i\sin 80^\circ}=$ __________ 。

【解答】1

【詳解】

$$\text{原式}=\frac{(-\cos 41^\circ+i\sin 41^\circ)(\cos 39^\circ-i\sin 39^\circ)}{-\cos 80^\circ+i\sin 80^\circ}=\frac{(\cos 41^\circ-i\sin 41^\circ)(\cos 39^\circ-i\sin 39^\circ)}{\cos 80^\circ-i\sin 80^\circ}$$

$$=\frac{[\cos(-41^\circ)+i\sin(-41^\circ)][\cos(-39^\circ)+i\sin(-39^\circ)]}{\cos(-80^\circ)+i\sin(-80^\circ)}$$

$$=\cos(-41^\circ-39^\circ+80^\circ)+i\sin(-41^\circ-39^\circ+80^\circ)=\cos 0^\circ+i\sin 0^\circ=1$$

3. 複數$z=\dfrac{1+i\tan\frac{\pi}{12}}{1-i\tan\frac{\pi}{12}}$，(1) z的極式為 __________ 。　(2) $z^6=$ __________ 。

【解答】(1) $\cos\frac{\pi}{6}+i\sin\frac{\pi}{6}$　(2) -1

【詳解】

$$\text{原式}=\frac{\cos\frac{\pi}{12}+i\sin\frac{\pi}{12}}{\cos\frac{\pi}{12}-i\sin\frac{\pi}{12}}=\frac{\cos\frac{\pi}{12}+i\sin\frac{\pi}{12}}{\cos(-\frac{\pi}{12})+i\sin(-\frac{\pi}{12})}=\cos(\frac{\pi}{12}+\frac{\pi}{12})+i\sin(\frac{\pi}{12}+\frac{\pi}{12})$$

$$=\cos\frac{\pi}{6}+i\sin\frac{\pi}{6}\Rightarrow z^6=(\cos\frac{\pi}{6}+i\sin\frac{\pi}{6})^6=\cos\pi+i\sin\pi=-1$$

4. $\sin 111^\circ+i\sin 201^\circ$化為極式為 __________ 。

【解答】$\cos 339^\circ+i\sin 339^\circ$

【詳解】

$$\sin 111^\circ+i\sin 201^\circ=\sin(90^\circ+21^\circ)+i\sin(180^\circ+21^\circ)=\cos 21^\circ-i\sin 21^\circ$$

$$=\cos(360^\circ-21^\circ)+i\sin(360^\circ-21^\circ)=\cos 339^\circ+i\sin 339^\circ$$

5. 設$z=\dfrac{(\cos 7^\circ+i\sin 7^\circ)^5(\cos 28^\circ+i\sin 28^\circ)^2}{\cos 61^\circ+i\sin 61^\circ}=\cos\theta+i\sin\theta$，$0^\circ\le\theta<360^\circ$。

(1) $\theta=$ __________ 。　(2)化為標準式$z=$ __________ 。

【解答】(1) $30°$　(2) $\frac{\sqrt{3}}{2}+\frac{1}{2}i$

【詳解】

(1) $z=\frac{(\cos 7°+i\sin 7°)^5(\cos 28°+i\sin 28°)^2}{\cos 61°+i\sin 61°}=\frac{(\cos 35°+i\sin 35°)(\cos 56°+i\sin 56°)}{\cos 61°+i\sin 61°}$

$= \cos(35°+56°-61°)+i\sin(35°+56°-61°)=\cos 30°+i\sin 30° \Rightarrow \theta=30°$

(2) $z=\cos 30°+i\sin 30°=\frac{\sqrt{3}}{2}+\frac{1}{2}i$

6. 複數$z=2\sqrt{3}+2i$的極式爲__________，其所代表點的極坐標爲__________。

【解答】$4(\cos\frac{\pi}{6}+i\sin\frac{\pi}{6})$；$[4，\frac{\pi}{6}]$

【詳解】

$z=2\sqrt{3}+2i$，$|z|=\sqrt{(2\sqrt{3})^2+2^2}=4$

∴　$z=4(\frac{\sqrt{3}}{2}+\frac{1}{2}i)=4(\cos\frac{\pi}{6}+i\sin\frac{\pi}{6})$，此爲 z 的極式，又 z 的極坐標爲$[4，\frac{\pi}{6}]$

7. 設$z\in C$，且$|z|=2|z-1|$，$\text{Arg}(\frac{z-1}{z})=\frac{\pi}{3}$，則$|z|=$__________。

【解答】$\frac{2\sqrt{3}}{3}$

【詳解】

∵　$|z|=2|z-1| \Rightarrow |\frac{z-1}{z}|=\frac{1}{2}$，而 $\text{Arg}(\frac{z-1}{z})=\frac{\pi}{3}$

∴　$\frac{z-1}{z}=\frac{1}{2}(\cos\frac{\pi}{3}+i\sin\frac{\pi}{3})=\frac{1+\sqrt{3}i}{4} \Rightarrow 4z-4=(1+\sqrt{3}i)z$

$\Rightarrow z(3-\sqrt{3}i)=4 \Rightarrow |z|\,|3-\sqrt{3}i|=|4| \Rightarrow |z|=\frac{4}{\sqrt{12}}=\frac{4}{2\sqrt{3}}=\frac{2}{\sqrt{3}}=\frac{2\sqrt{3}}{3}$

8. 求$\frac{(3+\sqrt{3}i)^6}{(1+\sqrt{3}i)^3}$之值 =__________。

【解答】216

【詳解】

∵　$3+\sqrt{3}i=2\sqrt{3}(\frac{\sqrt{3}}{2}+\frac{1}{2}i)=2\sqrt{3}(\cos\frac{\pi}{6}+i\sin\frac{\pi}{6})$

∴　$(3+\sqrt{3}i)^6=(2\sqrt{3})^6(\cos\pi+i\sin\pi)=-(2\sqrt{3})^6=-2^6\cdot 3^3$

∵　$1+\sqrt{3}i=2(\frac{1}{2}+\frac{\sqrt{3}}{2}i)=2(\cos\frac{\pi}{3}+i\sin\frac{\pi}{3})$　∴　$(1+\sqrt{3}i)^3=2^3(\cos\pi+i\sin\pi)=-2^3$

故$\frac{(3+\sqrt{3}i)^6}{(1+\sqrt{3}i)^3}=\frac{-2^6\cdot 3^3}{-2^3}=2^3\cdot 3^3=216$

9. 設$z=\dfrac{1+i}{1-\sqrt{3}i}$，求：(1)以主輻角將$z$化成的極式為＿＿＿＿＿＿。　(2) $z^{10}=$ ＿＿＿＿＿＿。

【解答】(1) $\dfrac{1}{\sqrt{2}}(\cos 105° + i\sin 105°)$　(2) $\dfrac{\sqrt{3}}{64}-\dfrac{1}{64}i$

【詳解】

(1) $|z| = |\dfrac{1+i}{1-\sqrt{3}i}| = \dfrac{|1+i|}{|1-\sqrt{3}i|} = \dfrac{\sqrt{2}}{\sqrt{1+3}} = \dfrac{\sqrt{2}}{2} = \dfrac{1}{\sqrt{2}}$

$\therefore\ z=\dfrac{1}{\sqrt{2}}(\dfrac{\sqrt{2}+\sqrt{2}i}{1-\sqrt{3}i})=\dfrac{1}{\sqrt{2}}[\dfrac{(\sqrt{2}+\sqrt{2}i)(1+\sqrt{3}i)}{(1-\sqrt{3}i)(1+\sqrt{3}i)}]=\dfrac{1}{\sqrt{2}}(\dfrac{\sqrt{2}+\sqrt{6}i+\sqrt{2}i-\sqrt{6}}{1+3})$

$=\dfrac{1}{\sqrt{2}}[-\dfrac{\sqrt{6}-\sqrt{2}}{4}+(\dfrac{\sqrt{6}+\sqrt{2}}{4})i]=\dfrac{1}{\sqrt{2}}(-\cos 75° + i\sin 75°)$

$=\dfrac{1}{\sqrt{2}}[\cos(180°-75°)+i\sin(180°-75°)]=\dfrac{1}{\sqrt{2}}(\cos 105° + i\sin 105°)$

(2) $z^{10}=(\dfrac{1}{\sqrt{2}})^{10}(\cos 105° + i\sin 105°)^{10}=\dfrac{1}{2^5}(\cos 1050° + i\sin 1050°)$

$=\dfrac{1}{32}[\cos(360°\times 3-30°)+i\sin(360°\times 3-30°)]$

$=\dfrac{1}{32}(\cos 30° - i\sin 30°)=\dfrac{1}{32}(\dfrac{\sqrt{3}}{2}-\dfrac{1}{2}i)=\dfrac{\sqrt{3}}{64}-\dfrac{1}{64}i$

10. $(1+\cos\dfrac{\pi}{3}+i\sin\dfrac{\pi}{3})^6=$ ＿＿＿＿＿＿。

【解答】-27

【詳解】

因為 $1+\cos\dfrac{\pi}{3}=2\cos^2\dfrac{\pi}{6}$，$\sin\dfrac{\pi}{3}=2i\sin\dfrac{\pi}{6}\cos\dfrac{\pi}{6}$

所以$(1+\cos\dfrac{\pi}{3}+i\sin\dfrac{\pi}{3})^6=(2\cos^2\dfrac{\pi}{6}+2i\sin\dfrac{\pi}{6}\cos\dfrac{\pi}{6})^6$

$=(2\cos\dfrac{\pi}{6})^6(\cos\dfrac{\pi}{6}+i\sin\dfrac{\pi}{6})^6=(\sqrt{3})^6(\cos\pi+i\sin\pi)=-27$