高雄市明誠中學 高一數學平時測驗 日期：95.04.04					
範圍	2-1,2 銳角三角函數	班級		姓名	
		座號			

一、選擇題 (每題 10 分)

1、(E) 化簡$\dfrac{1-\sin\theta}{\cos\theta}+\dfrac{\cos\theta}{1-\sin\theta}=$ (A)$\sin\theta$ (B)$2\cos\theta$ (C)$\tan\theta$ (D)$2\tan\theta$ (E)$2\sec\theta$

解析：

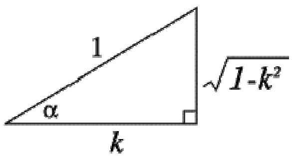
$$\frac{1-\sin\theta}{\cos\theta}+\frac{\cos\theta(1+\sin\theta)}{(1-\sin\theta)(1+\sin\theta)}=\frac{1-\sin\theta}{\cos\theta}+\frac{\cos\theta(1+\sin\theta)}{\cos^2\theta}=\frac{1-\sin\theta+1+\sin\theta}{\cos\theta}=\frac{2}{\cos\theta}=2\sec\theta$$

2、(A) 求$\sin 23^\circ\cos 67^\circ-\cos 23^\circ\csc 67^\circ+\cos^2 23^\circ$之值為 (A)0 (B)1 (C)2 (D)$\sin 23^\circ\cos 67^\circ$ (E)$\cos^2 23^\circ$

解析：原式$=\sin 23^\circ\cdot\sin 23^\circ-\cos 23^\circ\cdot\sec 23^\circ+\cos^2 23^\circ=\sin^2 23^\circ-1+\cos^2 23^\circ=0$

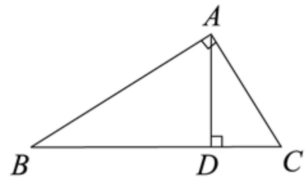
3、(B) 設$0^\circ<\alpha<90^\circ$，且$\cos\alpha=k$，則下列何者正確？ (A)$\sin\alpha=\dfrac{1}{k}$ (B)$\tan\alpha=\dfrac{\sqrt{1-k^2}}{k}$ (C)$\cot\alpha=\sqrt{k^2-1}$ (D)$\sec\alpha=\sqrt{1-k^2}$ (E)$\csc\alpha=\dfrac{k}{\sqrt{1-k^2}}$

解析：$\cos\alpha=k\Rightarrow$



$\Rightarrow\tan\alpha=\dfrac{\sqrt{1-k^2}}{k}$

4、(AB CD) 如圖所示，$\angle BAC=90^\circ$，$\overline{AD}\perp\overline{BC}$，則下列何者正確？（複選）
(A)$\overline{AD}=\overline{AB}\sin B$ (B)$\overline{BD}=\overline{AB}\cos B$ (C)$\overline{CD}=\overline{AC}\cos C$
(D)$\overline{AD}=\overline{BD}\tan B$ (E)$\overline{AC}=\overline{CD}\csc C$



解析：利用定義得知 (E)$\dfrac{\overline{AC}}{\overline{CD}}=\sec C$，故答案為(A)(B)(C)(D)。

二、填充題 (每題 10 分)

5、化簡下面各式：
(1)$\tan^4\theta-\sec^4\theta+2\sec^2\theta=$ ______ 。(2)$\cos^2\theta\sec^2(90^\circ-\theta)-\csc^2\theta=$ ______ 。

答案：(1) 1 (2) -1

解析：(1)原式$=(\tan^2)^2\theta-\sec^4\theta+2\sec^2\theta=(\sec^2\theta-1)^2-\sec^4\theta+2\sec^2\theta=1$
(2)原式$=\cos^2\theta\csc^2\theta-\csc^2\theta=\csc^2\theta(\cos^2\theta-1)=\csc^2\theta(-\sin^2\theta)=-1$

6、$\sin^2 28.5^\circ+\sin^2 34.5^\circ+\sin^2 61.5^\circ+\sin^2 55.5^\circ=$ ______ 。

答案：2

解析：原式$=\sin^2 28.5^\circ+\sin^2 34.5^\circ+\cos^2 28.5^\circ+\cos^2 34.5^\circ=1+1=2$

7、$\sin 30^\circ\tan 60^\circ\sec 45^\circ+\cos 45^\circ\cot 60^\circ=$ ______ 。

答案：$\dfrac{2\sqrt{6}}{3}$

解析：原式$=\dfrac{1}{2}\times\dfrac{\sqrt{3}}{1}\times\dfrac{\sqrt{2}}{1}+\dfrac{\sqrt{2}}{2}\times\dfrac{1}{\sqrt{3}}=\dfrac{\sqrt{6}}{2}+\dfrac{\sqrt{6}}{6}=\dfrac{2\sqrt{6}}{3}$

8、若θ爲銳角，$4\sin^2\theta-4\cos^2\theta=2+\sin\theta\cos\theta$，則$\tan\theta=$______，又$\sin\theta=$______。

答案：$2,\ \dfrac{2\sqrt{5}}{5}$

解析：同除以$\cos^2\theta\Rightarrow 4\cdot\dfrac{\sin^2\theta}{\cos^2\theta}-4=\dfrac{2}{\cos^2\theta}+\dfrac{\sin\theta}{\cos\theta}$

$4\tan^2\theta-4=2\sec^2\theta+\tan\theta\Rightarrow 4\tan^2\theta-4=2(\tan^2\theta+1)+\tan\theta$，

$(\tan\theta-2)(2\tan\theta+3)=0$，$\therefore\tan\theta=2$或$\tan\theta=-\dfrac{3}{2}$(不合)　$\therefore\sin\theta=\dfrac{2}{\sqrt{5}}=\dfrac{2\sqrt{5}}{5}$

9、設$0°<\angle A<18°,\ \ \sin A=\cos 5A$，則$\angle A=$______，又$\sec 4A=$______。

答案：$15°, 2$

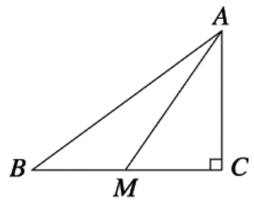
解析：$\sin A=\cos 5A\Rightarrow\angle A+5\angle A=90°\quad\therefore\angle A=15°,\ \ \sec 60°=2$

10、設$\tan x=\dfrac{1}{3}$，則$\dfrac{\sin x+2\cos x}{\cos x-\sin x}=$______。

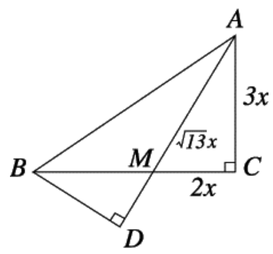
答案：$\dfrac{7}{2}$

解析：分子、分母同除以$\cos\theta\Rightarrow\dfrac{\tan x+2}{1-\tan x}=\dfrac{7}{2}$

11、如圖三角形ABC，$\angle C=90°$，M爲$\overline{BC}$之中點，設$\theta=\angle BAC$，已知$\sin\theta=\dfrac{4}{5}$，則$\tan(\angle BAM)=$______。



答案：$\dfrac{6}{17}$



解析：$\because\sin\theta=\dfrac{4}{5}\quad\therefore\overline{AB}:\overline{BC}:\overline{AC}=5:4:3$

$\because M$爲$\overline{BC}$中點　$\therefore\overline{AC}:\overline{CM}=3:2$

延長$\overline{AM}$，過B作$\overline{BD}\perp\overline{AM}$於$D$　$\Rightarrow\Delta BDM\sim\Delta ACM$

$\therefore\dfrac{\overline{BM}}{\overline{AM}}=\dfrac{\overline{BD}}{\overline{AC}}=\dfrac{\overline{DM}}{\overline{CM}}$，又$\dfrac{\overline{BM}}{\sqrt{13}}=\dfrac{\overline{BD}}{3}=\dfrac{\overline{DM}}{2}$

設$\overline{AC}=3x,\ \overline{CM}=2x, \overline{AM}=\sqrt{13}x$，$\overline{BM}=2x,\ \ \overline{BD}=\dfrac{6x}{\sqrt{13}},\ \overline{DM}=\dfrac{4x}{\sqrt{13}}$

$\therefore\tan(\angle BAM)=\dfrac{\dfrac{6}{\sqrt{13}}x}{\sqrt{13}x+\dfrac{4x}{\sqrt{13}}}=\dfrac{6}{17}$

12、設$\sin\theta+\cos\theta=\dfrac{7}{5}$，且$0°<\theta<45°$,則(1)$\sin\theta\cos\theta=$______, (2)$\sin\theta-\cos\theta=$______, (3)$\tan\theta+\cot\theta=$______, (4)$\sin^3\theta+\cos^3\theta=$______。

答案：(1)$\dfrac{12}{25}$　(2)$-\dfrac{1}{5}$　(3)$\dfrac{25}{12}$　(4)$\dfrac{91}{125}$

解析：$\sin\theta+\cos\theta=\dfrac{7}{5}\Rightarrow(\sin\theta+\cos\theta)^2=(\dfrac{7}{5})^2\quad\therefore 1+2\sin\theta\cdot\cos\theta=\dfrac{49}{25}\quad\therefore\sin\theta\cdot\cos\theta=\dfrac{12}{25}$,

$(\sin\theta-\cos\theta)^2=1-2\sin\theta\cdot\cos\theta=\frac{1}{25}\Rightarrow\sin\theta-\cos\theta=\pm\frac{1}{5}$

$\because 0^\circ<\theta<45^\circ\quad\therefore 0<\sin\theta<\cos\theta\quad\therefore\sin\theta-\cos\theta=-\frac{1}{5}$，

$\tan\theta+\cot\theta=\frac{1}{\sin\theta\cdot\cos\theta}=\frac{25}{12}$

$\sin^3\theta+\cos^3\theta=(\sin\theta+\cos\theta)(\sin^2\theta-\sin\theta\cos\theta+\cos^2\theta)$

$=(\sin\theta+\cos\theta)(1-\sin\theta\cos\theta)=\frac{7}{5}\times\frac{13}{25}=\frac{91}{125}$

13、若 $\sin A=\frac{\sqrt{3}}{2},\ \cos B=\frac{1}{2},\ \tan C=\frac{\sqrt{3}}{3},\ \sec D=\sqrt{2}$，且 $\theta=\angle A-\angle B-\angle C+2\angle D$，則 $\csc\theta=$_______。

答案：$\frac{2\sqrt{3}}{3}$

解析：$\angle A=60^\circ,\ \angle B=60^\circ,\ \angle C=30^\circ,\ \angle D=45^\circ,\quad\therefore\theta=60^\circ$，$\therefore\csc 60^\circ=\frac{2}{\sqrt{3}}=\frac{2\sqrt{3}}{3}$

13、設 θ 爲銳角，$\sin\theta,\ \cos\theta$ 爲 $x^2-(6k-1)x+k=0$ 的二根，則 $k=$______。

答案：$\frac{7}{18}$

解析：$\sin\theta+\cos\theta=6k-1,\ \sin\theta\cos\theta=k$，又 $(\sin\theta+\cos\theta)^2=1+2\sin\theta\cdot\cos\theta$

$\therefore(6k-1)^2=1+2k\quad\therefore k=0$ (不合)或 $\frac{7}{18}$

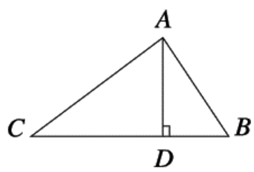
14、若 $\sin\theta=\cos^2\theta$，則 $\frac{1}{1-\sin\theta}+\frac{1}{1+\sin\theta}=$______。

答案：$\sqrt{5}+1$

解析：$\sin\theta=\cos^2\theta\Rightarrow\sin\theta=1-\sin^2\theta,\ \therefore\sin\theta=\frac{-1\pm\sqrt{5}}{2}$ （$\frac{-1-\sqrt{5}}{2}$ 不合）

$\therefore$ 原式 $=\frac{1+\sin\theta+1-\sin\theta}{1-\sin^2\theta}=\frac{2}{\cos^2\theta}=\frac{2}{\sin\theta}=\frac{2}{\frac{-1+\sqrt{5}}{2}}=\frac{4}{\sqrt{5}-1}\times\frac{\sqrt{5}+1}{\sqrt{5}+1}=\sqrt{5}+1$

15、如圖 $\triangle ABC$ 中，$\overline{AD}$ 爲 $\overline{BC}$ 邊的高，$\tan B=\frac{3}{2},\ \sin C=\frac{3}{5}$，又 $\overline{BC}=24$，則(1) $\overline{AD}=$______。 (2) $\overline{AB}=$______。



答案：(1)12　(2)$4\sqrt{13}$

解析：由 $\tan B=\frac{3}{2}$，設 $\overline{AD}=3x,\overline{BD}=2x,\overline{AB}=\sqrt{13}x$，又 $\sin C=\frac{3}{5}$，$\therefore$

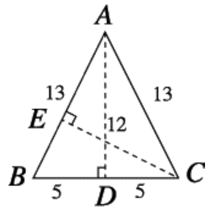
$\overline{CD}=4x$

$\therefore 2x+4x=24\quad x=4,\quad\therefore\overline{AB}=4\sqrt{13}$

16、等腰$\triangle ABC$中，$\overline{AB}=\overline{AC}=13,\ \overline{BC}=10$，則$\sin C=$______，又$\sin A=$______。

答案：$\dfrac{12}{13},\ \dfrac{120}{169}$

解析：過A作$\overline{AD}\perp\overline{BC}$於$D\Rightarrow\overline{BD}=\overline{CD}=5,\overline{AD}=12$，$\therefore\sin C=\dfrac{12}{13}$



過C作$\overline{CE}\perp\overline{AB}$於$E$，$\Delta ABC=\dfrac{1}{2}\times\overline{BC}\times\overline{AD}=\dfrac{1}{2}\times\overline{AB}\times\overline{CE}$

$\Rightarrow\overline{CE}=\dfrac{\overline{BC}\times\overline{AD}}{\overline{AB}}=\dfrac{10\times12}{13}\quad\therefore\sin A=\dfrac{120}{169}$

17、化簡$\dfrac{1}{1+\sin20^\circ}+\dfrac{1}{1+\cos20^\circ}+\dfrac{1}{1+\tan20^\circ}+\dfrac{1}{1+\cot20^\circ}+\dfrac{1}{1+\sec20^\circ}+\dfrac{1}{1+\csc20^\circ}=$______。

答案：3

解析：原式$=(\dfrac{1}{1+\sin20^\circ}+\dfrac{1}{1+\csc20^\circ})+(\dfrac{1}{1+\cos20^\circ}+\dfrac{1}{1+\sec20^\circ})+(\dfrac{1}{1+\tan20^\circ}+\dfrac{1}{1+\cot20^\circ})$

$=\dfrac{1+\csc20^\circ+1+\sin20^\circ}{1+\sin20^\circ+\csc20^\circ+1}+\dfrac{1+\sec20^\circ+1+\cos20^\circ}{1+\cos20^\circ+\sec20^\circ+1}+\dfrac{1+\cot20^\circ+1+\tan20^\circ}{1+\tan20^\circ+\cot20^\circ+1}$

$=1+1+1=3$

18、化簡下列各式：

(1) $(\sin\theta-\csc\theta)^2+(\sec\theta-\cos\theta)^2-(\tan\theta+\cot\theta)^2=$______。

(2) $(1+\tan\theta-\sec\theta)(1+\cot\theta+\csc\theta)=$______。

答案：(1) -3　(2) 2

解析：

(1)

$\sin^2\theta-2\sin\theta\csc\theta+\csc^2\theta+\sec^2\theta-2\sec\theta\cos\theta+\cos^2\theta-\tan^2\theta-2\tan\theta\cot\theta-\cot^2\theta$

$=1+1+1-2-2-2=-3$

(2) $(\dfrac{\cos\theta+\sin\theta-1}{\cos\theta})(\dfrac{\sin\theta+\cos\theta+1}{\sin\theta})=\dfrac{2\sin\theta\cos\theta}{\sin\theta\cos\theta}=2$

19、求下列各式之値

(1) $\sin^2 37^\circ+\sin^2 53^\circ=$______。

(2) $\sin^2 10^\circ+\sin^2 20^\circ+\sin^2 30^\circ+\sin^2 40^\circ+\sin^2 50^\circ+\sin^2 60^\circ+\sin^2 70^\circ+\sin^2 80^\circ=$______。

答案：(1) 1　(2) 4

解析：(1)原式$=\sin^2 37^\circ+\cos^2 37^\circ=1$

(2) $\sin^2 10^\circ+\sin^2 80^\circ=\sin^2 10^\circ+\cos^2 10^\circ=1$

同理$\sin^2 20^\circ+\sin^2 70^\circ=1$，$\sin^2 30^\circ+\sin^2 60^\circ=1$，$\sin^2 40^\circ+\sin^2 50^\circ=1$，故所求 4。

20、設$x^2+(\tan\theta+\cot\theta)x+1=0$有一根爲$2-\sqrt5$，則$\sin\theta\cos\theta=$______，又另一根爲______。

答案：$\dfrac{\sqrt5}{10},\ -(2+\sqrt5)$

解析：一根爲$2-\sqrt5$，另一根爲β　$\therefore$兩根積$(2-\sqrt5)\beta=1$　$\therefore\beta=-(2+\sqrt5)$

又兩根和$-(\tan\theta+\cot\theta)=2-\sqrt5+\beta=-2\sqrt5\Rightarrow\tan\theta+\cot\theta=2\sqrt5$

$\therefore\sin\theta\cos\theta=\dfrac{1}{\tan\theta+\cot\theta}=\dfrac{1}{2\sqrt5}=\dfrac{\sqrt5}{10}$

21、設$\angle A$爲銳角，若$\sec^2\theta = 3\tan\theta - 1$，則$\sin A =$ ______ 或______ 。

答案：$\dfrac{\sqrt{2}}{2},\ \dfrac{2}{\sqrt{5}}$

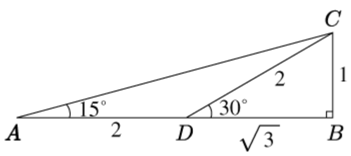
解析：$\tan^2\theta + 1 = 3\tan\theta - 1$，設$\tan\theta = t \Rightarrow \quad \therefore t^2 + 1 = 3t - 1 \quad \therefore t = 1$或 2

$\therefore \tan\theta = 1, 2 \Rightarrow \sin A = \dfrac{\sqrt{2}}{2}$或$\dfrac{2}{\sqrt{5}}$

22、有一梯子靠牆斜放與地面成$60°$角，若梯長 6 公尺，則梯腳距離牆角多遠？

答案：

$$\cos 60° = \frac{\overline{AC}}{\overline{AB}} \Rightarrow \overline{AC} = \overline{AB} \cdot \cos 60° = 6 \times \frac{1}{2} = 3\text{（公尺）}$$



23、設△ABC 中，$\angle B$ 爲直角，D 點在$\overline{AB}$上，而$\overline{AD} = \overline{DC}$，$\angle BDC = 30°$，爲了方便，設$\overline{BC} = 1$。試利用此圖形求$\sin 15°,\ \cos 15°,\ \tan 15°$之值。

答案：由$\angle A + \angle ACD = \angle CDA$，且$\overline{AD} = \overline{DC}$，$\angle A = \angle ACD = \dfrac{1}{2}\angle BDC = \dfrac{1}{2} \cdot 30° = 15°$

在△BCD 中，$\angle B$ 爲直角，$\angle BDC = 30°$，設$\overline{BC} = 1$，$\overline{DB} = \sqrt{3}$，$\overline{DC} = 2$，$\overline{AD} = \overline{DC} = 2$

$\overline{AB} = 2 + \sqrt{3}$，且$\overline{AC}^2 = (2+\sqrt{3})^2 + 1^2 = 8 + 2 \cdot 2\sqrt{3} = 8 + 2\sqrt{12} = (\sqrt{6} + \sqrt{2})^2$

所以$\overline{AC} = \sqrt{6} + \sqrt{2}$，故

$$\sin 15° = \sin A = \frac{\overline{BC}}{\overline{AC}} = \frac{1}{\sqrt{6} + \sqrt{2}} = \frac{\sqrt{6} - \sqrt{2}}{4}$$

$$\sin 15° = \sin A = \frac{\overline{AD}}{\overline{AC}} = \frac{2 + \sqrt{3}}{\sqrt{6} + \sqrt{2}} = \frac{(2+\sqrt{3})(\sqrt{6} - \sqrt{2})}{4} = \frac{\sqrt{6} + \sqrt{2}}{4}$$

$$\tan 15° = \frac{1}{2 + \sqrt{3}} = 2 - \sqrt{3}$$

24、試證$\dfrac{1 + \sin\theta + \cos\theta}{1 + \sin\theta - \cos\theta} + \dfrac{1 + \sin\theta - \cos\theta}{1 + \sin\theta + \cos\theta} = 2\csc\theta$。

答案：通分，左式$= \dfrac{(1 + \sin\theta + \cos\theta)^2 + (1 + \sin\theta - \cos\theta)^2}{(1 + \sin\theta)^2 - (\cos\theta)^2} = \dfrac{2[(1 + \sin\theta)^2 + \cos^2\theta]}{1 + 2\sin\theta + \sin^2\theta - \cos^2\theta}$

$= \dfrac{2(2 + 2\sin\theta)}{2\sin\theta(1 + \sin\theta)} = \dfrac{2}{\sin\theta} = 2\csc\theta =$右式