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一、填充題(每題 10 分)

1. (1) $(\log_2 9 + \log_8 3)(\log_3 16 + \log_9 4) = \underline{\hspace{2cm}}$.
 (2) $\log_3 54 + \log_3 4.5 + \log_3 \frac{1}{27\sqrt{3}} - \log_3 \sqrt[3]{81} = \underline{\hspace{2cm}}$.

答案：(1) $\frac{35}{3}$ (2) $\frac{1}{6}$

解析：(1)原式 = $(\log_2 3^2 + \frac{\log_2 3}{\log_2 8})(\log_3 16 + \log_3 2)$
 $= (2\log_2 3 + \frac{1}{3}\log_2 3)(\log_3 32)$
 $= (\frac{7}{3}\log_2 3) \cdot (\log_3 2^5)$
 $= (\frac{7}{3}\log_2 3)(5\log_3 2)$
 $= \frac{35}{3}(\log_2 3)(\log_3 2) = \frac{35}{3} \cdot 1 = \frac{35}{3}$

(2)原式 = $\log_3 \frac{54 \times 4.5 \times \frac{1}{27\sqrt{3}}}{3\sqrt[3]{3}}$
 $= \log_3 \frac{9}{3\sqrt[3]{3}} = \log_3 \frac{3}{\sqrt{3} \cdot \sqrt[3]{3}}$
 $= \log_3 \frac{3}{3^{\frac{1}{2} + \frac{1}{3}}} = \log_3 3^{\frac{1}{6}} = \frac{1}{6}$

2. 化簡下列各式：

- (1) $(\frac{1}{2})^{-1+\log_{0.5} 4} = \underline{\hspace{2cm}}$.
 (2) $\log(\log \sqrt[10]{10}) = \underline{\hspace{2cm}}$.
 (3) $3^{1+\log_3 4} = \underline{\hspace{2cm}}$.

答案：(1) 8 (2) -1 (3) 12

解析：(1)原式 = $(\frac{1}{2})^{\frac{\log_1 2 + \log_1 4}{2}} = (\frac{1}{2})^{\frac{\log_1 8}{2}} = 8$

(2)原式 = $\log(\log 10^{\frac{1}{10}}) = \log \frac{1}{10} = -1$

(3)原式 = $3^{\log_3 3 + \log_3 4} = 3^{\log_3 12} = 12 = 1$

3. 試求下列各值：

- (1)若 $\log_3 x = 4$ ，則 $x = \underline{\hspace{2cm}}$.
 (2)若 $\log_{10} y = \frac{1}{3}$ ，則 $y = \underline{\hspace{2cm}}$.
 (3)若 $\log_z 9 = 2$ ，則 $z = \underline{\hspace{2cm}}$.

(4)若 $\log_a 6 = -\frac{1}{2}$, 則 $a =$ _____.

(5)若 $\log_{\sqrt{2}} b = 8$, 則 $b =$ _____.

(6)若 $\log_c \sqrt[4]{3} = 2$, 則 $c =$ _____.

答案：(1)81 (2) $\sqrt[3]{10}$ (3)3 (4) $\frac{1}{36}$ (5)16 (6) $\sqrt[8]{3}$

解析：(1) $3^4 = x = 81$

(2) $y = 10^{\frac{1}{3}} = \sqrt[3]{10}$

(3) $z^2 = 9 \Rightarrow z = 3$

(4) $a^{-\frac{1}{2}} = 6 \Rightarrow a^{-1} = 36 \Rightarrow a = \frac{1}{36}$

(5) $b = \sqrt{2}^8 = 2^4 = 16$

(6) $c^2 = \sqrt[4]{3} \Rightarrow c = \sqrt[8]{3}$

4. 化簡：(1) $\log_3 49 \cdot \log_7 81 =$ _____.(2) $\log_4 9 \cdot \log_3 625 \cdot \log_5 16 =$ _____.

答案：(1)8 (2)16

解析：(1)原式 = $\log_3 7^2 \cdot \log_7 81 = 2 \cdot \log_3 7 \cdot \log_7 81 = 2 \cdot \log_3 81 = 2 \times 4 = 8$

(2)原式 = $\log_2 3 \cdot \log_3 5^4 \cdot \log_5 16 = \log_2 3 \cdot (4 \log_3 5) \cdot \log_5 16 = 4 \cdot \log_2 5 \cdot \log_5 16 = 4 \cdot \log_2 16 = 4 \times 4 = 16$

5. (1) $3^{\log_3 5 + \log_3 2} =$ _____.

(2) $8^{-\log_2 7} =$ _____.

(3) $3^{\log_{\sqrt{3}} 4} =$ _____.

(4) $0.1^{\log_{10} 3} =$ _____.

答案：(1)10 (2) $\frac{1}{343}$ (3)16 (4) $\frac{1}{3}$

解析：(1)原式 = $3^{\log_3 10} = 10$

(2)原式 = $2^{-3 \log_2 7} = 2^{\log_2 7^{-3}} = 7^{-3} = \frac{1}{343}$

(3)原式 = $3^{\log_3 16} = 16$

(4)原式 = $0.1^{\log_{0.1} \frac{1}{3}} = \frac{1}{3}$

6. 若 $a = \log_5 2$, $b = \log_{50} 4$, 則 $\frac{a-b}{ab} =$ _____.

答案： $\frac{1}{2}$

解析： $\frac{a-b}{ab} = \frac{1}{b} - \frac{1}{a} = \log_4 50 - \log_2 5 = \log_4 50 - \log_4 25 = \log_4 \frac{50}{25} = \log_4 2 = \frac{1}{2}$

7. 解： $5^{2 \log_5 3} = 3x + 4$, 則 $x =$ _____.

答案： $\frac{5}{3}$

解析： $5^{\log_5 9} = 9 = 3x + 4 \Rightarrow 3x = 5 \Rightarrow x = \frac{5}{3}$

8. 設 a, b, c 為正整數，若 $a \log_{520} 2 + b \log_{520} 5 + c \log_{520} 13 = 3$, 則 $a + b + c =$ _____。

答案：15

解析： $a \log_{520} 2 + b \log_{520} 5 + c \log_{520} 13 = 3$
 $\Rightarrow \log_{520} 2^a + \log_{520} 5^b + \log_{520} 13^c = 3$
 $\Rightarrow \log_{520} 2^a \cdot 5^b \cdot 13^c = 3 \Rightarrow 520^3 = 2^a \cdot 5^b \cdot 13^c$
 $\Rightarrow (2^3 \cdot 5 \cdot 13)^3 = 2^9 \cdot 5^3 \cdot 13^3 = 2^a \cdot 5^b \cdot 13^c$ 得 $a=9, b=3, c=3$ ，故 $a+b+c=9+3+3=15$

9. $(\log_6 3)^2 + \frac{\log_6 18}{\log_2 6} = \underline{\hspace{2cm}}$.

答案： 1

解析： 原式 $= (\log_6 3)^2 + \frac{\log_6 3 + \log_6 6}{\log_2 6}$
 $= (\log_6 3)^2 + (\log_6 3 + 1) \cdot \log_6 2$
 $= (\log_6 3)^2 + (\log_6 3)(\log_6 2) + \log_6 2$
 $= (\log_6 3)[\log_6 3 + \log_6 2] + \log_6 2$
 $= (\log_6 3) \cdot (\log_6 6) + \log_6 2 = \log_6 3 + \log_6 2 = \log_6 6 = 1$

10. 化简： $25^{\log_5 3} + \sqrt{6^{\frac{1}{2} \log_6 4}} = \underline{\hspace{2cm}}$.

答案： $9 + \sqrt{2}$

解析： 原式 $= 25^{\log_5 3} + \sqrt{6^{\log_6 4^{\frac{1}{2}}}} = 9 + \sqrt{6^{\log_6 2}} = 9 + \sqrt{6^{\log_{\sqrt{6}} \sqrt{2}}} = 9 + \sqrt{2}$

11. 试求下列各值：

(1) $\log_4 \frac{1}{3} \cdot \log_5 \frac{1}{4} \cdot \log_6 \frac{1}{5} \cdot \log_7 \frac{1}{6} \cdot \log_8 \frac{1}{7} \cdot \log_9 \frac{1}{8} = \underline{\hspace{2cm}}$.

(2) $\log_4 9 \cdot \log_3 125 \cdot \log_5 8 = \underline{\hspace{2cm}}$.

答案： (1) $\frac{1}{2}$ (2) 9

解析： (2) 原式 $= (\log_9 8^{-1})(\log_8 7^{-1})(\log_7 6^{-1})(\log_6 5^{-1})(\log_5 4^{-1})(\log_4 3^{-1})$
 $= (-\log_9 8)(-\log_8 7)(-\log_7 6)(-\log_6 5)(-\log_5 4)(-\log_4 3)$
 $= \log_9 8 \cdot \log_8 7 \cdot \log_7 6 \cdot \log_6 5 \cdot \log_5 4 \cdot \log_4 3 = \log_9 3 = \frac{1}{2}$

(3) 原式 $= \log_4 3^2 \cdot \log_3 5^3 \cdot \log_5 2^3$
 $= (2 \log_4 3)(3 \log_3 5)(3 \log_5 2)$
 $= 18(\log_4 3 \cdot \log_3 5 \cdot \log_5 2) = 18 \log_4 2 = 18 \cdot \frac{1}{2} = 9$

12. $\log 2 \cdot \log 50 - \log 5 \cdot \log 20 - \log 4 = \underline{\hspace{2cm}}$.

答案： -1

解析： 原式 $= \log 2 \cdot (\log 5 + \log 10) - \log 5(\log 2 + \log 10) - 2 \log 2$
 $= \log 2 \cdot \log 5 + \log 2 - \log 5 \cdot \log 2 - \log 5 - 2 \log 2$
 $= -\log 5 - \log 2 = -(\log 5 + \log 2) = -\log 10 = -1$.

13. 化简： $\log_3 \sqrt{3\sqrt{7}+6} + \log_3 \sqrt{3\sqrt{7}-6} = \underline{\hspace{2cm}}$.

答案： $\frac{3}{2}$

解析： 原式 $= \log_3 \sqrt{(3\sqrt{7}+6)(3\sqrt{7}-6)} = \log_3 \sqrt{63-36} = \log_3 27^{\frac{1}{2}} = \frac{1}{2} \log_3 27 = \frac{3}{2}$

14. 化簡： $\log_a a^3 + \log_a \frac{1}{a^2} - \log_a \sqrt[3]{a} + \log_a \frac{1}{\sqrt{a}} = \underline{\hspace{2cm}}$.

答案： $\frac{1}{6}$

解析：原式 = $\log_a (a^3 \times a^{-2} \times a^{-\frac{1}{3}} \times a^{-\frac{1}{2}}) = \log_a a^{\frac{1}{6}} = \frac{1}{6}$

15. 化簡： $(\log 2)[(\log_{\frac{1}{2}} \frac{1}{4})^{-1} + (\log_{\sqrt{5}} 2)^{-1}] = \underline{\hspace{2cm}}$.

答案： $\frac{1}{2}$

解析：原式 = $(\log 2)[2^{-1} + \frac{1}{\log_{\sqrt{5}} 2}]$
 $= (\log 2)(\frac{1}{2} + \log_2 \sqrt{5})$
 $= (\log 2) \cdot (\log_2 \sqrt{2} + \log_2 \sqrt{5})$
 $= (\log 2) \cdot (\log_2 \sqrt{10}) = \log \sqrt{10} = \frac{1}{2}$

17. 若 $\log_6 2 = a, \log_6 7 = b$ ，試以 a, b 表示 $\log_{112} 42 = \underline{\hspace{2cm}}$.

答案： $\frac{1+b}{4a+b}$

解析： $\log_{112} 42 = \frac{\log_6 42}{\log_6 112} = \frac{\log_6 (6 \times 7)}{\log_6 (16 \times 7)} = \frac{\log_6 6 + \log_6 7}{\log_6 2^4 + \log_6 7} = \frac{1+b}{4a+b}$

18. 若 $\log_2 3 = a$ ，試以 a 表示 (1) $\log_2 1296 = \underline{\hspace{2cm}}$ (2) $\log_2 \frac{27}{4} = \underline{\hspace{2cm}}$

(3) $\frac{1}{2} \log_3 5 - \log_3 \frac{\sqrt{5}}{2} = \underline{\hspace{2cm}}$ (4) $3^x = 4^5, x = \underline{\hspace{2cm}}$.

答案：(1) $4+4a$ (2) $3a-2$ (3) $\frac{1}{a}$ (4) $\frac{10}{a}$

解析：(1) $\log_2 1296 = \log_2 (2^4 \times 3^4) = \log_2 2^4 + \log_2 3^4 = 4 + 4 \log_2 3 = 4 + 4a$

(2) $\log_2 \frac{27}{4} = \log_2 27 - \log_2 4 = \log_2 3^3 - 2 = 3 \log_2 3 - 2 = 3a - 2$

(3) $\frac{1}{2} \log_3 5 - \log_3 \frac{\sqrt{5}}{2} = \log_3 \sqrt{5} - \log_3 \frac{\sqrt{5}}{2} = \log_3 \frac{\sqrt{5}}{\frac{\sqrt{5}}{2}} = \log_3 2 = \frac{1}{\log_2 3} = \frac{1}{a}$

(4) $x = \log_3 4^5 = 5 \log_3 4 = 5 \log_3 2^2 = 10 \log_3 2 = 10 \frac{1}{\log_2 3} = \frac{10}{a}$