<table>
<tr><td colspan="6">高雄市明誠中學 高二數學平時測驗　　日期：102.11.04</td></tr>
<tr><td rowspan="2">範圍</td><td rowspan="2">1-4.5 和、差、測量</td><td>班級</td><td>二年____班</td><td rowspan="2">姓名</td><td rowspan="2"></td></tr>
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一、填充題　（每題 10 分 ）

1. 設 $f(x)=3\sin^2 2x+4\cos^2 x\,,0^\circ\le x\le 180^\circ$ ，則 $f(x)$ 之最大值＝________.

答案：$\dfrac{16}{3}$

解析：$f(x)=3\sin^2 2x+4\cos^2 x=3(1-\cos^2 2x)+4\cdot\dfrac{1+\cos 2x}{2}$

$=3-3\cos^2 2x+2+2\cos 2x$

$=-3(\cos^2 2x-\dfrac{2}{3}\cos 2x)+5$

$=-3(\cos 2x-\dfrac{1}{3})^2+5+\dfrac{1}{3}=-3(\cos 2x-\dfrac{1}{3})^2+\dfrac{16}{3}$　　$\therefore$ 當 $\cos 2x=\dfrac{1}{3}$ 時，有最大值 $=\dfrac{16}{3}$

2. 試比較大小：$a=2\sin 20^\circ\cos 20^\circ\,,b=2\cos^2 22.5^\circ-1\,,c=\dfrac{2\tan 35^\circ}{1-\tan^2 35^\circ}$.答：________.

答案：$a<b<c$

解析：$a=\sin 40^\circ\,,b=\cos 45^\circ=\sin 45^\circ\,,c=\tan 70^\circ>\tan 45^\circ=1$　　$\therefore a<b<c$

3. 若 $\tan\dfrac{\theta}{2}=2$ ，則 $\tan(\theta+45^\circ)=$__________ ， $\dfrac{6\sin\theta+\cos\theta}{3\sin\theta-2\cos\theta}=$__________.

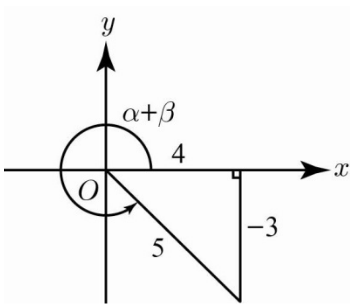
答案：$-\dfrac{1}{7},\ \dfrac{7}{6}$

解析：$\tan\theta=\tan(2\cdot\dfrac{\theta}{2})=\dfrac{2\cdot 2}{1-4}=-\dfrac{4}{3}$　；　$\tan(\theta+45^\circ)=\dfrac{1+(-\frac{4}{3})}{1-1\times(-\frac{4}{3})}=\dfrac{-\frac{1}{3}}{\frac{7}{3}}=-\dfrac{1}{7}$

$\dfrac{6\sin\theta+\cos\theta}{3\sin\theta-2\cos\theta}=\dfrac{6\tan\theta+1}{3\tan\theta-2}=\dfrac{6(-\frac{4}{3})+1}{3(-\frac{4}{3})-2}=\dfrac{-24+3}{-12-6}=\dfrac{-21}{-18}=\dfrac{7}{6}$

4. 已知 $270^\circ<\alpha+\beta<360^\circ\,,90^\circ<\alpha-\beta<180^\circ\,,\cos(\alpha+\beta)=\dfrac{4}{5}$,

$\cos(\alpha-\beta)=-\dfrac{4}{5}$ ，則 $\cos 2\alpha=$__________ ， $\cos 2\beta=$__________.



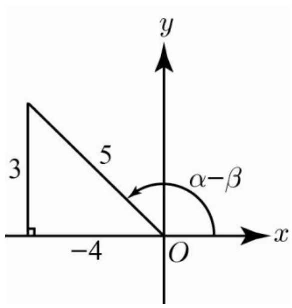
答案：$-\dfrac{7}{25},\ -1$

解析：

$\cos(\alpha+\beta)=\dfrac{4}{5}$　　　　$\sin(\alpha+\beta)=-\dfrac{3}{5}$

$\cos(\alpha-\beta)=-\dfrac{4}{5}$　　　　$\sin(\alpha-\beta)=\dfrac{3}{5}$

$\cos 2\alpha=\cos[(\alpha+\beta)+(\alpha-\beta)]=\dfrac{4}{5}(-\dfrac{4}{5})-(-\dfrac{3}{5})\dfrac{3}{5}=\dfrac{-16+9}{25}=\dfrac{-7}{25}$



$$\cos 2\beta=\cos[(\alpha+\beta)-(\alpha-\beta)]=\frac{4}{5}(-\frac{4}{5})+(-\frac{3}{5})\frac{3}{5}=\frac{-16-9}{25}=-1$$

5. 若α,β,γ為銳角，且$\tan\alpha=\frac{1}{8}$, $\tan\beta=\frac{1}{5}$, $\tan\gamma=\frac{1}{2}$，則$\alpha+\beta+\gamma=$__________.

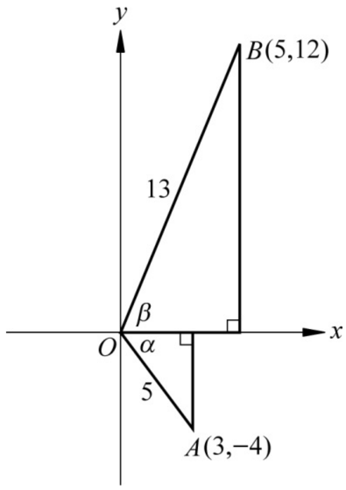
答案：$45°$

解析：$\tan(\alpha+\beta)=\dfrac{\frac{1}{8}+\frac{1}{5}}{1-\frac{1}{8}\cdot\frac{1}{5}}=\dfrac{\frac{13}{40}}{\frac{39}{40}}=\frac{1}{3}$ $\tan[(\alpha+\beta)+\gamma]=\dfrac{\frac{1}{3}+\frac{1}{2}}{1-\frac{1}{3}\cdot\frac{1}{2}}=\dfrac{\frac{5}{6}}{\frac{5}{6}}=1$

又$0°<\alpha<45°$, $0°<\beta<45°$, $0°<\gamma<45°\Rightarrow 0°<\alpha+\beta+\gamma<135°$ $\therefore\alpha+\beta+\gamma=45°$

6. 坐標平面上，O表原點，$A(3,-4)$, $B(5,12)$，則

(1) $\cos\angle AOB=$________；(2) $\sin\angle AOB=$________.



答案：(1) $\frac{63}{65}$ (2) $\frac{16}{65}$

解析：

$\because\sin\alpha=\frac{-4}{5},\cos\alpha=\frac{3}{5},\sin\beta=\frac{12}{13},\cos\beta=\frac{5}{13}$

$\therefore\cos\angle AOB=\cos(\alpha+\beta)=\frac{3}{5}\cdot\frac{5}{13}-(\frac{-4}{5})\cdot\frac{12}{13}=\frac{15+48}{65}=\frac{63}{65}$

$\sin\angle AOB=\sin(\alpha+\beta)=(\frac{-4}{5})\cdot\frac{5}{13}+\frac{3}{5}\cdot\frac{12}{13}=\frac{-20+36}{65}=\frac{16}{65}$

7. $\sin16°\cdot\cos316°-\sin224°\cdot\cos344°=$________.

答案：$\frac{\sqrt{3}}{2}$

解析：原式$=\sin16°\cos44°-(-\sin44°)\cos16°$

$=\sin16°\cos44°+\cos16°\sin44°=\sin60°=\frac{\sqrt{3}}{2}$

8. $\cos24°\cos48°\cos96°\cos192°=$________.

答案：$\frac{1}{16}$

解析：令$a=\cos24°\cos48°\cos96°\cos192°$

$\Rightarrow\sin24°\cdot a=\sin24°\cos24°\cos48°\cos96°\cos192°$

$=\frac{1}{2}\sin48°\cos48°\cos96°\cos192°$

$=\frac{1}{4}\sin96°\cos96°\cos192°$

$=\frac{1}{8}\sin192°\cos192°$

$=\frac{1}{16}\sin384°=\frac{1}{16}\sin24°\Rightarrow a=\frac{1}{16}$

9. 設$\tan\alpha$與$\tan\beta$為$x^2-4x-2=0$之二根，則

(1) $\tan(\alpha+\beta)=$________. (2) $\tan(2\alpha+2\beta)=$________.

答案：(1) $\frac{4}{3}$ (2) $\frac{24}{-7}$

解析：$\begin{cases}\tan\alpha+\tan\beta=4\\ \tan\alpha\cdot\tan\beta=-2\end{cases}$

$(1)\tan(\alpha+\beta)=\dfrac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}=\dfrac{4}{1-(-2)}=\dfrac{4}{3}$

$(2)\tan(2\alpha+2\beta)=\tan[2(\alpha+\beta)]=\dfrac{2\tan(\alpha+\beta)}{1-\tan^2(\alpha+\beta)}=\dfrac{\frac{4}{3}\times 2}{1-\frac{16}{9}}=\dfrac{\frac{8}{3}}{\frac{-7}{9}}=\dfrac{24}{-7}$

10. 設 $270°<\theta<360°$，若 $\sin\theta+\cos\theta=\dfrac{1}{5}$，則(1) $\sin 2\theta=$ ________. (2) $\cos\theta=$ ________.

答案：$(1)-\dfrac{24}{25}$　$(2)\dfrac{4}{5}$

解析：$(1)\sin\theta+\cos\theta=\dfrac{1}{5}\Rightarrow 1+2\sin\theta\cos\theta=\dfrac{1}{25}\Rightarrow\sin 2\theta=-\dfrac{24}{25}$

$(2)(\cos\theta-\sin\theta)^2=1-2\sin\theta\cos\theta=1-(-\dfrac{24}{25})=\dfrac{49}{25}$　$\Rightarrow\cos\theta-\sin\theta=\pm\dfrac{7}{5}$（負不合）

又 $\cos\theta+\sin\theta=\dfrac{1}{5}\Rightarrow\cos\theta=\dfrac{4}{5}$

11. $\triangle ABC$ 中， $\tan B=1,\tan C=2,\overline{AC}=10$，則 $\overline{BC}=$ ________.

答案：$6\sqrt{5}$

解析：$\because\tan B=1\Rightarrow\angle B=45°$

$\tan A=\tan[180°-(B+C)]=-\tan(B+C)=-\dfrac{1+2}{1-1\cdot 2}=3\Rightarrow\sin A=\dfrac{3}{\sqrt{10}}$

設 $\overline{BC}=a$　$\therefore\dfrac{a}{\frac{3}{\sqrt{10}}}=\dfrac{10}{\frac{\sqrt{2}}{2}}\Rightarrow a=6\sqrt{5}$

12. 已知 $\sin\beta=\dfrac{1}{4},\sin(\alpha+\beta)=1$，則 $\sin(2\alpha+\beta)=$ ________.

答案：$\dfrac{1}{4}$

解析：$\because\sin(\alpha+\beta)=1$

$\therefore\alpha+\beta=360°\cdot n+90°$，$n$ 為整數

$\Rightarrow\sin(2\alpha+\beta)=\sin[2(\alpha+\beta)-\beta]=\sin[(720°\cdot n+180°)-\beta]=\sin(180°-\beta)=\sin\beta=\dfrac{1}{4}$

13. 已知 $\dfrac{1+\tan\theta}{1-\tan\theta}=2$，則 $\sin 2\theta+\sin^2\theta+\cos 2\theta=$ ________.

答案：$\dfrac{3}{2}$

解析：$1+\tan\theta=2-2\tan\theta\Rightarrow 3\tan\theta=1\Rightarrow\tan\theta=\dfrac{1}{3}$

$\sin 2\theta+\sin^2\theta+\cos 2\theta=\dfrac{2\tan\theta}{1+\tan^2\theta}+\sin^2\theta+(\cos^2\theta-\sin^2\theta)=\dfrac{2\cdot\frac{1}{3}}{1+\frac{1}{9}}+\cos^2\theta$

$$=\frac{\frac{2}{3}}{\frac{10}{9}}+\frac{1}{\frac{1}{\cos^2\theta}}=\frac{18}{30}+\frac{1}{1+\tan^2\theta}=\frac{3}{5}+\frac{1}{1+\frac{1}{9}}=\frac{3}{5}+\frac{9}{10}=\frac{15}{10}=\frac{3}{2}$$

14. 若α,β為銳角，且$\cos\alpha=\frac{1}{17},\cos(\alpha+\beta)=-\frac{47}{51}$，則$\cos\beta=$________.

答案：$\frac{1}{3}$

解析：$\sin\alpha=\frac{12\sqrt{2}}{17}$　又$0^\circ<\alpha+\beta<180^\circ$

$\sqrt{51^2-47^2}=\sqrt{98\times 4}=14\sqrt{2}$　$\therefore\sin(\alpha+\beta)=\frac{14\sqrt{2}}{51}$

$\cos\beta=\cos[(\alpha+\beta)-\alpha]=(-\frac{47}{51})\cdot\frac{1}{17}+\frac{14\sqrt{2}}{51}\times\frac{12\sqrt{2}}{17}=\frac{-47+336}{51\times 17}=\frac{289}{51\times 17}=\frac{1}{3}$.

15. 若$\frac{\sin(\alpha+\beta)}{\sin(\alpha-\beta)}=\frac{1}{2}$，則$\frac{\tan\alpha}{\tan\beta}=$________.

答案：-3

解析：$\because\frac{\sin(\alpha+\beta)}{\sin(\alpha-\beta)}=\frac{\sin\alpha\cos\beta+\cos\alpha\sin\beta}{\sin\alpha\cos\beta-\cos\alpha\sin\beta}=\frac{\frac{\sin\alpha}{\cos\alpha}+\frac{\sin\beta}{\cos\beta}}{\frac{\sin\alpha}{\cos\alpha}-\frac{\sin\beta}{\cos\beta}}=\frac{\tan\alpha+\tan\beta}{\tan\alpha-\tan\beta}=\frac{1}{2}$

$\therefore 2\tan\alpha+2\tan\beta=\tan\alpha-\tan\beta\Rightarrow\tan\alpha=-3\tan\beta$　故$\frac{\tan\alpha}{\tan\beta}=-3$

16. 設$x^2+px+q=0$之兩根為$\sin\theta,\cos\theta$，試求$2\sin^2\frac{\theta}{2}(\cos\frac{\theta}{2}-\sin\frac{\theta}{2})^2=$________.

答案：$1+p+q$

解析：$\because\sin\theta+\cos\theta=-p$　$\sin\theta\cos\theta=q$

$2\sin^2\frac{\theta}{2}(\cos\frac{\theta}{2}-\sin\frac{\theta}{2})^2=2\cdot\frac{1-\cos\theta}{2}\cdot(\cos^2\frac{\theta}{2}-2\sin\frac{\theta}{2}\cos\frac{\theta}{2}+\sin^2\frac{\theta}{2})$

$=(1-\cos\theta)(1-\sin\theta)=1-(\sin\theta+\cos\theta)+\sin\theta\cos\theta=1-(-p)+q=1+p+q$

17. 若$\tan\frac{\theta}{2}=3$，求$\cos 2\theta+\sin 2\theta=$________.

答案：$\frac{-17}{25}$

解析：$\tan\theta=\frac{2\tan\frac{\theta}{2}}{1-\tan^2\frac{\theta}{2}}=\frac{2\times 3}{1-9}=\frac{6}{-8}=-\frac{3}{4}$　$\cos 2\theta+\sin 2\theta=\frac{1-\tan^2\theta}{1+\tan^2\theta}+\frac{2\tan\theta}{1+\tan^2\theta}$

$$=\frac{1-(-\frac{3}{4})^2+2\cdot(-\frac{3}{4})}{1+(-\frac{3}{4})^2}=\frac{1-\frac{9}{16}-\frac{3}{2}}{1+\frac{9}{16}}=\frac{\frac{16-9-24}{16}}{\frac{25}{16}}=\frac{-17}{25}$$

18. $f(x)=8x^3-6x+1$除以$x-\cos 320^\circ$之餘式為________.

答案：0

解析：根據餘式定理

$f(x)\div(x-\cos 320°)$ 之餘式 $\Rightarrow x=\cos 320°=\cos 40°$ 代入

$f(\cos 40°)=8\cos^3 40°-6\cos 40°+1=2(4\cos^3 40°-3\cos 40°)+1$

$=2\cos 120°+1=2\cdot(-\frac{1}{2})+1=0$

19. 若 $360°<\theta<540°$，且 $\sin\theta=\frac{1}{7}$，則 $\sin\frac{\theta}{2}+\cos\frac{\theta}{2}=$__________.

答案：$-\frac{2\sqrt{14}}{7}$

解析：$\because \sin\theta=\frac{1}{7}$

$\therefore (\sin\frac{\theta}{2}+\cos\frac{\theta}{2})^2=\sin^2\frac{\theta}{2}+2\sin\frac{\theta}{2}\cos\frac{\theta}{2}+\cos^2\frac{\theta}{2}=1+\sin\theta=1+\frac{1}{7}=\frac{8}{7}$

又 $360°<\theta<540°\Rightarrow 180°<\frac{\theta}{2}<270°\quad\Rightarrow \sin\frac{\theta}{2}+\cos\frac{\theta}{2}<0$

$\therefore \sin\frac{\theta}{2}+\cos\frac{\theta}{2}=-\sqrt{\frac{8}{7}}=-\frac{2\sqrt{14}}{7}$

20. 試求：(1) $\sin 18°=$________；(2) $\cos 36°=$________.

答案：(1) $\frac{\sqrt{5}-1}{4}$ (2) $\frac{\sqrt{5}+1}{4}$

解析：(1) $\sin 18°$

令 $\theta=18°\Rightarrow 5\theta=90°\Rightarrow 3\theta=90°-2\theta$

$\Rightarrow \sin 3\theta=\sin(90°-2\theta)$

$\Rightarrow 3\sin\theta-4\sin^3\theta=\cos 2\theta=1-2\sin^2\theta$

$\Rightarrow 4\sin^3\theta-2\sin^2\theta-3\sin\theta+1=0$ (令 $t=\sin\theta$)

$\Rightarrow 4t^3-2t^2-3t+1=0$

考慮一次有理因式檢驗定理

$(t\pm1)(2t\pm1)(4t\pm1)$ 可能為上式一次有理因式 $\therefore f(1)=0$

$\Rightarrow (t-1)(4t^2+2t-1)=0\Rightarrow t=1,\frac{-2\pm\sqrt{4+16}}{2\cdot 4}$

$\Rightarrow \sin\theta=\sin 18°=1,\frac{-1\pm\sqrt{5}}{4}\qquad\therefore \sin 18°=\frac{\sqrt{5}-1}{4}$

(2) $\cos 36°$

$\Rightarrow \theta=36°\Rightarrow 5\theta=180°\Rightarrow 3\theta=180°-2\theta$

$\Rightarrow \cos 3\theta=\cos(180°-2\theta)=-\cos 2\theta$

$\Rightarrow 4\cos^3\theta-3\cos\theta=-(2\cos^2\theta-1)$

$\Rightarrow 4\cos^3\theta+2\cos^2\theta-3\cos\theta-1=0$

$\Rightarrow 4y^3+2y^2-3y-1=0,$

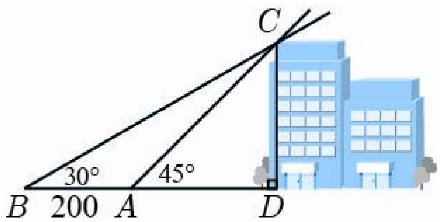
考慮一次有理因式檢驗定理

$(y\pm1)\cdot(2y\pm1)\cdot(4y\pm1)$ 可能為上式一次有理因式 $\therefore f(-1)=0$

$\Rightarrow (y+1)(4y^2-2y-1)=0\Rightarrow y=-1,\frac{2\pm\sqrt{4+16}}{8}$

$\Rightarrow \cos\theta=\cos 36°=-1,\frac{1\pm\sqrt{5}}{4}>0\qquad\therefore \cos 36°=\frac{\sqrt{5}+1}{4}$

21. 如圖，在 A 點測得大樓頂端 C 點之仰角為 $45°$，往後退 200 m 到達 B 點，測得 C 點之仰角為 $30°$，試求大樓的高度 $\overline{CD}=$____.



答案：$100(\sqrt{3}+1)$

解析：設 $\overline{CD}=x$，又 $\angle ACD=45° \Rightarrow \overline{AD}=x$

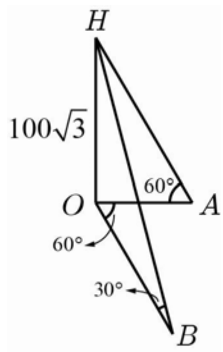
$$\because \frac{\overline{CD}}{\overline{BD}}=\tan 30° \Rightarrow \frac{x}{200+x}=\frac{1}{\sqrt{3}} \Rightarrow \sqrt{3}x=x+200$$

$$\Rightarrow (\sqrt{3}-1)x=200 \Rightarrow x=\frac{200}{\sqrt{3}-1}=\frac{200(\sqrt{3}+1)}{(\sqrt{3}-1)(\sqrt{3}+1)}=\frac{200(\sqrt{3}+1)}{2}=100(\sqrt{3}+1)$$

22. 地面上有一塔，其高為 $100\sqrt{3}$，地面上有 A,B 兩點，點 A 在塔之正東，點 B 在塔的東 $60°$ 南，今有 1 人由塔頂測得 A,B 之俯角各為 $60°$ 及 $30°$，則 A,B 兩點之距離為________.

答案：$100\sqrt{7}$

解析：



若塔高 $h \Rightarrow \overline{OA}=\frac{h}{\sqrt{3}}, \overline{OB}=\sqrt{3}h$

$$\therefore \cos 60°=\frac{(\frac{h}{\sqrt{3}})^2+(\sqrt{3}h)^2-\overline{AB}^2}{2\cdot\frac{h}{\sqrt{3}}\sqrt{3}h}=\frac{1}{2} \Rightarrow h^2=\frac{h^2}{3}+3h^2-\overline{AB}^2$$

$$\Rightarrow \overline{AB}^2=\frac{7}{3}h^2=\frac{7}{3}\times 10000\times 3 \qquad \therefore \overline{AB}=100\sqrt{7}$$

23. 某湖邊上有二點 A,B，今某人站於 C 處，測出 $\angle ACB=60°, \overline{AC}=15, \overline{BC}=20$，則 $\overline{AB}=$_____.

答案：$5\sqrt{13}$

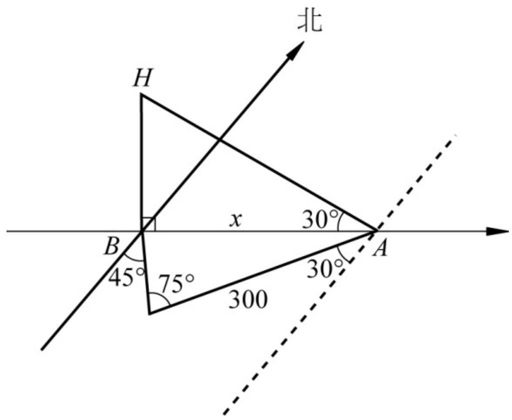
解析：$\overline{AB}^2=\overline{AC}^2+\overline{BC}^2-2\overline{AC}\cdot\overline{BC}\cos 60°=225+400-2\cdot 15\cdot 20\cdot\frac{1}{2}=625-300=325$

$\overline{AB}=\sqrt{325}=5\sqrt{13}$

24. 平面上有 A、B 兩點，A 在塔的正東，B 在塔的東南且在 A 的南 $30°$ 西 300 公尺處，在 A 測得塔頂的仰角為 $30°$，則塔高為________公尺.

答案：$50(3+\sqrt{3})$

解析：



$$\because \frac{x}{\sin 75°}=\frac{300}{\sin 45°}$$

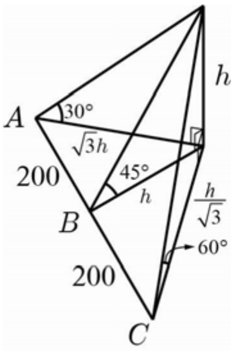
$$\therefore x=\frac{300}{\frac{1}{\sqrt{2}}}\cdot\frac{\sqrt{6}+\sqrt{2}}{4}=\frac{300\cdot(2+2\sqrt{3})}{4}=150(\sqrt{3}+1)$$ 故

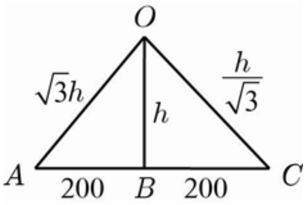
$$\overline{OH}=\frac{150(\sqrt{3}+1)}{\sqrt{3}}=50(3+\sqrt{3})$$

25. 從地面上共線 A,B,C 三點各測得一山之仰角依序為 $30°, 45°, 60°$，若 $\overline{AB}=200$ m, $\overline{BC}=200$ m，則山高＝________.

答案：$100\sqrt{6}$

解析：



$\Rightarrow$


$\Rightarrow (\sqrt{3}h)^2 + (\frac{h}{\sqrt{3}})^2 = 2(h^2 + 40000)$

$\Rightarrow 3h^2 + \frac{h^2}{3} = 2h^2 + 80000 \Rightarrow \frac{4}{3}h^2 = 80000 \Rightarrow h^2 = 60000 \Rightarrow h = 100\sqrt{6}$

26. 某日，甲、乙、丙三人分別位於 A, B, C 三點觀測一山，發現三人之仰角皆為 $30°$ ，且 $\overline{AB} = 700, \overline{BC} = 800, \overline{CA} = 500$ ，則山高＝________.

答案：$\frac{700}{3}$

解析：$\triangle ABC = \sqrt{1000 \cdot 300 \cdot 200 \cdot 500} = 100000\sqrt{3} = \frac{abc}{4R} = \frac{800 \times 700 \times 500}{4R} \Rightarrow R = \frac{700}{\sqrt{3}}$

$h = \frac{\frac{700}{\sqrt{3}}}{\sqrt{3}} = \frac{700}{3}$