高雄市明誠中學 高三數學平時測驗					日期：102.03.21
範圍	Chap1 數列與級數、極限	班級	三年____班	姓名	
		座號			

一、填充題 (每題 10 分)

1.若數列$\langle(\dfrac{3x}{2x+1})^n\rangle$收斂，則 x 之範圍為____________ .

解答 $-\dfrac{1}{5}<x\le 1$

解析 $|\dfrac{3x}{2x+1}|<1$或$\dfrac{3x}{2x+1}=1$，

①$(3x)^2<(2x+1)^2 \Rightarrow 5x^2-4x-1<0 \Rightarrow (x-1)(5x+1)<0 \Rightarrow -\dfrac{1}{5}<x<1$，

②$3x=2x+1 \Rightarrow x=1$，

由①②可知，x 之範圍為$-\dfrac{1}{5}<x\le 1$.

2.級數$\sum\limits_{n=1}^{\infty}(3x+1)^n$ 收斂，試求 x 的範圍為____________ .

解答 $-\dfrac{2}{3}<x<0$

解析 依題意：$|3x+1|<1 \Rightarrow -1<3x+1<1 \Rightarrow -\dfrac{2}{3}<x<0$.

3.求$\lim\limits_{n\to\infty}\dfrac{(n+1)(n+2)(n+3)}{(n^2+n+1)(2n+1)}=$____________ .

解答 $\dfrac{1}{2}$

解析 原式$=\lim\limits_{n\to\infty}\dfrac{n^3+6n^2+11n+6}{2n^3+3n^2+3n+1}=\lim\limits_{n\to\infty}\dfrac{1+\frac{6}{n}+\frac{11}{n^2}+\frac{6}{n^3}}{2+\frac{3}{n}+\frac{3}{n^2}+\frac{1}{n^3}}=\dfrac{1}{2}$.

4.求$\lim\limits_{n\to\infty}\dfrac{5^{n+2}+7^{n+2}}{5^n+7^n}=$____________ .

解答 49

解析 $\lim\limits_{n\to\infty}\dfrac{5^{n+2}+7^{n+2}}{5^n+7^n}=\lim\limits_{n\to\infty}\dfrac{5^n\cdot 5^2+7^n\cdot 7^2}{5^n+7^n}=\lim\limits_{n\to\infty}\dfrac{25\cdot(\frac{5}{7})^n+49}{(\frac{5}{7})^n+1}=49$.

5.已知$a_n=\dfrac{1^2}{n^3}+\dfrac{2^2}{n^3}+\cdots+\dfrac{n^2}{n^3}$，求$\lim\limits_{n\to\infty}a_n=$____________ .

解答 $\dfrac{1}{3}$

解析　$a_n=\dfrac{1^2+2^2+3^2+\cdots+n^2}{n^3}=\dfrac{\frac{1}{6}n(n+1)(2n+1)}{n^3}=\dfrac{2n^3+3n^2+n}{6n^3}$

$\therefore \lim\limits_{n\to\infty}a_n=\lim\limits_{n\to\infty}\dfrac{2+\frac{3}{n}+\frac{1}{n^2}}{6}=\dfrac{1}{3}$.

6.求$\lim\limits_{n\to\infty}(\dfrac{n^2-n}{n+1}-\dfrac{n^2+n}{n-1})=$____________ .

解答　-4

解析　原式$=\lim\limits_{n\to\infty}\dfrac{(n^2-n)(n-1)-(n^2+n)(n+1)}{(n+1)(n-1)}=\lim\limits_{n\to\infty}\dfrac{-4n^2}{n^2-1}=\lim\limits_{n\to\infty}\dfrac{-4}{1-\frac{1}{n^2}}=-4$.

7.將$4.21\overline{7}$化成分數為____________ .

解答　$4\dfrac{49}{225}$

解析　$4.21\overline{7}=4+0.21\overline{7}=4+\dfrac{217-21}{900}=4+\dfrac{196}{900}=4\dfrac{49}{225}$.

8.設無窮等比級數$1+\dfrac{1}{5}+\dfrac{1}{5^2}+\cdots+\dfrac{1}{5^{n-1}}+\cdots$的和為$S$，前$n$項部分的和為$S_n$，若$|S-S_n|<\dfrac{1}{1000}$，則最小正整數$n=$____________ .

解答　5

解析　$S_n=\dfrac{a_1(1-r^n)}{1-r}=\dfrac{1[1-(\frac{1}{5})^n]}{1-\frac{1}{5}}=\dfrac{1-(\frac{1}{5})^n}{\frac{4}{5}}=\dfrac{5}{4}-\dfrac{1}{4\cdot5^{n-1}}$，$S=\dfrac{a_1}{1-r}=\dfrac{1}{1-\frac{1}{5}}=\dfrac{5}{4}$，

$|S-S_n|<\dfrac{1}{1000}\Rightarrow|\dfrac{5}{4}-\dfrac{5}{4}+\dfrac{1}{4\cdot5^{n-1}}|<\dfrac{1}{1000}\Rightarrow\dfrac{1}{4\cdot5^{n-1}}<\dfrac{1}{1000}$，當$n=5$時，

$|S-S_n|=\dfrac{1}{2500}<\dfrac{1}{1000}$，故$n=5$.

9.求無窮級數$1+\dfrac{2}{5}+\dfrac{3}{5^2}+\dfrac{4}{5^3}+\cdots+\dfrac{n}{5^{n-1}}+\cdots=$____________ .

解答　$\dfrac{25}{16}$

解析　令$S=1+\dfrac{2}{5}+\dfrac{3}{5^2}+\dfrac{4}{5^3}+\cdots+\dfrac{n}{5^{n-1}}+\cdots$，

將「 = 」兩邊同乘$\dfrac{1}{5}$，得$\dfrac{1}{5}S=\dfrac{1}{5}+\dfrac{2}{5^2}+\dfrac{3}{5^3}+\dfrac{4}{5^4}+\cdots+\dfrac{n}{5^n}+\cdots$，

將兩式相減，得$\dfrac{4}{5}S=1+\dfrac{1}{5}+\dfrac{1}{5^2}+\dfrac{1}{5^3}+\cdots+\dfrac{1}{5^n}+\cdots=\dfrac{1}{1-\frac{1}{5}}=\dfrac{5}{4}$，所以$S=\dfrac{25}{16}$.

10.設 a 與 b 均為實數．若 $\frac{a}{2}+\frac{b}{2^2}+\frac{a}{2^3}+\frac{b}{2^4}+\cdots+\frac{a}{2^{2n-1}}+\frac{b}{2^{2n}}+\cdots=3$，則 $2a+b=$ ____________ ．

解答　9

解析　因為 $\frac{a}{2}+\frac{b}{2^2}+\frac{a}{2^3}+\frac{b}{2^4}+\cdots+\frac{a}{2^{2n-1}}+\frac{b}{2^{2n}}+\cdots$

$$=a(\frac{1}{2}+\frac{1}{2^3}+\cdots)+b(\frac{1}{2^2}+\frac{1}{2^4}+\cdots)=a\cdot\frac{\frac{1}{2}}{1-\frac{1}{4}}+b\cdot\frac{\frac{1}{4}}{1-\frac{1}{4}}=\frac{4}{3}(\frac{a}{2}+\frac{b}{4})=\frac{1}{3}(2a+b)\text{，}$$

所以 $\frac{1}{3}(2a+b)=3$，即 $2a+b=9$ ．

11.試求：(1) $\lim\limits_{n\to\infty}(\frac{99}{100})^n=$ ____________ ．　(2)設 $a>0$，求 $\lim\limits_{n\to\infty}\frac{a+(0.7799)^n}{2a+(0.8888)^n}=$ ____________ ．

解答　(1)0;(2)$\frac{1}{2}$

解析　(1)0．(2)原式 $=\frac{a}{2a}=\frac{1}{2}$ ．

12.設數列 $\langle a_n\rangle$ 滿足 $3n^2+2<(n^2+1)a_n<3n^2+7$，則 $\lim\limits_{n\to\infty}a_n=$ ____________ ．

解答　3

解析　$\because 3n^2+2<(n^2+1)a_n<3n^2+7$

$\therefore \frac{3n^2+2}{n^2+1}<a_n<\frac{3n^2+7}{n^2+1}$

而 $\lim\limits_{n\to\infty}\frac{3n^2+2}{n^2+1}=\lim\limits_{n\to\infty}\frac{3n^2+7}{n^2+1}=3$

$\therefore \lim\limits_{n\to\infty}a_n=3$ ．

13.令 $a=\lim\limits_{n\to\infty}\frac{4^n+5^n}{(-6)^n}$， $b=\lim\limits_{n\to\infty}\frac{6^{n+10}-6\cdot 3^{2n}}{5^{n+3}+3^{2n+1}}$，則數對$(a,b)=$ ____________ ．

解答　$(0,-2)$

解析　$a=\lim\limits_{n\to\infty}\frac{(-\frac{4}{6})^n+(-\frac{5}{6})^n}{1}=\frac{0+0}{1}=0$，

$$b=\lim_{n\to\infty}\frac{6^{10}\cdot 6^n-6\cdot 9^n}{5^3\cdot 5^n+3^1\cdot 9^n}=\lim_{n\to\infty}\frac{6^{10}(\frac{6}{9})^n-6}{125(\frac{5}{9})^n+3}=\frac{0-6}{0+3}=-2\ .$$

14.求 $\lim\limits_{n\to\infty}(\frac{2+3}{5}+\frac{2^2+3^2}{5^2}+\cdots+\frac{2^n+3^n}{5^n})=$ ____________ ．

解答　$\frac{13}{6}$

解析　原式$=\lim_{n\to\infty}[(\frac{2}{5}+(\frac{2}{5})^2+\cdots+(\frac{2}{5})^n)+(\frac{3}{5}+(\frac{3}{5})^2+\cdots+(\frac{3}{5})^n)]$

$$=\lim_{n\to\infty}[(\frac{\frac{2}{5}(1-(\frac{2}{5})^n)}{1-\frac{2}{5}})+(\frac{\frac{3}{5}(1-(\frac{3}{5})^n)}{1-\frac{3}{5}})]=\frac{\frac{2}{5}}{\frac{3}{5}}+\frac{\frac{3}{5}}{\frac{2}{5}}=\frac{2}{3}+\frac{3}{2}=\frac{13}{6}\ .$$

15.無窮級數$\frac{2}{3}-\frac{4}{9}+\frac{8}{27}-\frac{16}{81}+\cdots+(-1)^{n-1}\cdot(\frac{2}{3})^n+\cdots=$____________ .

解答　$\frac{2}{5}$

解析　原式$=\frac{a_1}{1-r}=\frac{\frac{2}{3}}{1-(-\frac{2}{3})}=\frac{\frac{2}{3}}{\frac{5}{3}}=\frac{2}{5}$.

16.試求$\lim_{n\to\infty}\frac{1+4+\cdots+(3n-2)}{1+3+\cdots+(2n-1)}=$____________ .

解答　$\frac{3}{2}$

解析　利用等差求和，原式$=\lim_{n\to\infty}\frac{\frac{n[1+(3n-2)]}{2}}{\frac{n[1+(2n-1)]}{2}}=\lim_{n\to\infty}\frac{3n-1}{2n}=\frac{3}{2}$.

17.設$\lim_{n\to\infty}\frac{an^2+bn+4}{n-2}=2$，求$(a,b)=$____________ .

解答　$(0,2)$

解析　$a=0$，$b=2$，$\therefore(a,b)=(0,2)$.

18.級數$\sum_{n=1}^{\infty}2(-\frac{1}{3})^{n-1}$的和為____________ .

解答　$\frac{3}{2}$

解析　公比為$-\frac{1}{3}$，首項為$2(-\frac{1}{3})^0=2$, $\therefore$和$=\frac{2}{1-(-\frac{1}{3})}=\frac{3}{2}$.

19.試求$\lim_{n\to\infty}\frac{(\frac{1}{3})^n+(\frac{1}{4})^n}{(\frac{1}{3})^{n+1}+(\frac{1}{4})^{n+1}}=$____________ .

解答　3

解析 分子、分母同乘以 3^n，得 $\lim\limits_{n\to\infty}\dfrac{1+(\frac{3}{4})^n}{\frac{1}{3}+(\frac{3}{4})^n\cdot\frac{1}{4}}=3$.

20.$1-\dfrac{1}{3}+\dfrac{1}{2}-\dfrac{1}{9}+\dfrac{1}{4}-\dfrac{1}{27}+\dfrac{1}{8}-\dfrac{1}{81}+\cdots=$＿＿＿＿＿＿＿＿ .

解答 $\dfrac{3}{2}$

解析 所求 $=(1+\dfrac{1}{2}+\dfrac{1}{4}+\dfrac{1}{8}+\cdots)-(\dfrac{1}{3}+\dfrac{1}{9}+\dfrac{1}{27}+\dfrac{1}{81}+\cdots)=\dfrac{1}{1-\frac{1}{2}}-\dfrac{\frac{1}{3}}{1-\frac{1}{3}}=\dfrac{3}{2}$.

21.已知 $\sum\limits_{n=1}^{\infty}r^{n-1}=\dfrac{3r}{r-1}$，則實數 r 的值為＿＿＿＿＿＿＿＿ .

解答 $-\dfrac{1}{3}$

解析 $\sum\limits_{n=1}^{\infty}r^{n-1}=\dfrac{3r}{r-1}\Rightarrow\begin{cases}-1<r<1\\ \dfrac{1}{1-r}=\dfrac{3r}{r-1}\end{cases}\Rightarrow r=-\dfrac{1}{3}$ 或 $r=1$（不合） .

22.求下列各極限值：

(1) $\lim\limits_{n\to\infty}\dfrac{3^n+4^n}{3^{n+1}+4^{n+1}}=$＿＿＿＿＿＿＿＿ . (2) $\lim\limits_{n\to\infty}\dfrac{1-2\cdot10^n}{3+5\cdot10^n}=$＿＿＿＿＿＿＿＿ .

(3) $\lim\limits_{n\to\infty}\dfrac{a^{n+1}+b^{n+1}}{a^n+b^n}=$＿＿＿＿＿＿＿＿（但 $a>0$，$b>0$） .

解答 (1) $\dfrac{1}{4}$;(2) $-\dfrac{2}{5}$;(3) a 或 b

解析 (1) $\lim\limits_{n\to\infty}\dfrac{3^n+4^n}{3^{n+1}+4^{n+1}}=\lim\limits_{n\to\infty}\dfrac{3^n+4^n}{3^n\cdot3+4^n\cdot4}=\lim\limits_{n\to\infty}\dfrac{(\frac{3}{4})^n+1}{(\frac{3}{4})^n\cdot3+4}=\dfrac{1}{4}$.

(2) $\lim\limits_{n\to\infty}\dfrac{1-2\cdot10^n}{3+5\cdot10^n}=\lim\limits_{n\to\infty}\dfrac{\frac{1}{10^n}-2}{\frac{3}{10^n}+5}=-\dfrac{2}{5}$.

(3)①當 $a>b$ 時，有 $0<\dfrac{b}{a}<1$，$\therefore\lim\limits_{n\to\infty}(\dfrac{b}{a})^n=0$，故 $\lim\limits_{n\to\infty}\dfrac{a^{n+1}+b^{n+1}}{a^n+b^n}=\lim\limits_{n\to\infty}\dfrac{a+b(\frac{b}{a})^n}{1+(\frac{b}{a})^n}=a$，

②當 $a<b$ 時，有 $0<\frac{a}{b}<1$，∴ $\lim\limits_{n\to\infty}(\frac{a}{b})^n=0$，故 $\lim\limits_{n\to\infty}\frac{a^{n+1}+b^{n+1}}{a^n+b^n}=\lim\limits_{n\to\infty}\frac{a(\frac{a}{b})^n+b}{(\frac{a}{b})^n+1}=b$，

③當 $a=b$ 時，$\lim\limits_{n\to\infty}\frac{a^{n+1}+b^{n+1}}{a^n+b^n}=\lim\limits_{n\to\infty}\frac{2a^{n+1}}{2a^n}=\lim\limits_{n\to\infty}a=a$，

∴所求為 a 或 b．

23.化簡 $0.4\overline{32}+0.\overline{210}=$ ____________．

解答 $0.6\overline{425334}$

解析 $[2,3]=6$，新數的循環節應有 6 位，

所求 $=0.4323232\cdots+0.2102102\cdots=0.6\overline{425334}$．

24.若數列 $\langle a_n\rangle$ 滿足 $\lim\limits_{n\to\infty}\frac{2a_n+5}{4a_n-7}=3$，則 $\lim\limits_{n\to\infty}a_n=$ ____________．

解答 $\frac{13}{5}$

解析 令 $\lim\limits_{n\to\infty}a_n=k$（$k\in\mathbb{R}$），

原式 $\Rightarrow\frac{2k+5}{4k-7}=3\Rightarrow k=\frac{13}{5}$，∴ $\lim\limits_{n\to\infty}a_n=\frac{13}{5}$．

25.若首項為 a，公比為 0.01 的無窮等比級數和等於循環小數 $1.\overline{2}$，則 $a=$ ____________．

解答 1.21

解析 $\frac{a}{1-0.01}=1.\overline{2}=1\frac{2}{9}=\frac{11}{9}\Rightarrow\frac{a}{0.99}=\frac{11}{9}\Rightarrow\frac{a}{0.11}=\frac{11}{1}\Rightarrow a=0.11\times 11=1.21$．

26.求 $\lim\limits_{n\to\infty}\frac{1}{\sqrt{n}}(\frac{1}{\sqrt{n}+\sqrt{n+2}}+\frac{1}{\sqrt{n+1}+\sqrt{n+3}}+\cdots+\frac{1}{\sqrt{2n-1}+\sqrt{2n+1}})=$ ____________．

解答 $\sqrt{2}-1$

解析

$$\lim_{n\to\infty}\frac{1}{\sqrt{n}}(\frac{1}{\sqrt{n}+\sqrt{n+2}}+\frac{1}{\sqrt{n+1}+\sqrt{n+3}}+\cdots+\frac{1}{\sqrt{2n-1}+\sqrt{2n+1}})$$

$$=\lim_{n\to\infty}\frac{1}{\sqrt{n}}(\frac{\sqrt{n+2}-\sqrt{n}}{2}+\frac{\sqrt{n+3}-\sqrt{n+1}}{2}+\cdots+\frac{\sqrt{2n+1}-\sqrt{2n-1}}{2})$$

$$=\lim_{n\to\infty}\frac{1}{2\sqrt{n}}[(\cancel{\sqrt{n+2}}-\sqrt{n})+(\cancel{\sqrt{n+3}}-\sqrt{n+1})+(\cancel{\sqrt{n+4}}-\cancel{\sqrt{n+2}})+(\cancel{\sqrt{n+5}}-\cancel{\sqrt{n+3}})+\cdots$$

$$+(\sqrt{2n}-\cancel{\sqrt{2n-2}})+(\sqrt{2n+1}-\cancel{\sqrt{2n-1}})]$$

$$=\lim_{n\to\infty}\frac{1}{2\sqrt{n}}(\sqrt{2n}+\sqrt{2n+1}-\sqrt{n}-\sqrt{n+1})=\frac{2\sqrt{2}-2}{2}=\sqrt{2}-1$$．