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範圍	2-3 方程式(C)	班級	一年____班	姓名
		座號		

一、填充題 (每題 10 分)

1. 若  $\alpha = 1+i$ ,  $\beta = 2-3i$ , 求

(1)  $\alpha + \beta =$  \_\_\_\_\_ . (2)  $\alpha - \beta =$  \_\_\_\_\_ . (3)  $\alpha\beta =$  \_\_\_\_\_ . (4)  $\frac{\alpha}{\beta} =$  \_\_\_\_\_ .

**解答** (1)  $3-2i$ ; (2)  $-1+4i$ ; (3)  $5-i$ ; (4)  $\frac{-1+5i}{13}$

**解析** (1)  $\alpha + \beta = (1+i) + (2-3i) = (1+2) + (1-3)i = 3-2i$

(2)  $\alpha - \beta = (1+i) - (2-3i) = 1+i-2+3i = -1+4i$

(3)  $\alpha\beta = (1+i)(2-3i) = 2-3i+2i-3i^2 = 5-i$

(4)  $\frac{\alpha}{\beta} = \frac{1+i}{2-3i} = \frac{(1+i)(2+3i)}{(2-3i)(2+3i)} = \frac{2+3i+2i+3i^2}{4-9i^2} = \frac{-1+5i}{13}$

2. 設  $b, c$  皆為整數, 若方程式  $x^4 + 2x^3 + bx^2 + cx + 8 = 0$  有四個相異有理根, 則此方程式之最小有理根為\_\_\_\_\_.

**解答** -4

**解析**  $\because b, c$  皆為整數, 且  $x^4$  項係數為 1  $\Rightarrow$  四根為相異整數,  
 $\therefore x^4 + 2x^3 + bx^2 + cx + 8 = (x+1)(x-1)(x+4)(x-2) \Rightarrow$  最小有理根為 -4 .

3. 複數  $(-2 + \sqrt{3}i)^4$  的(1)實部為\_\_\_\_\_ . (2)虛部為\_\_\_\_\_ .

**解答** (1) -47; (2)  $-8\sqrt{3}$

**解析**  $(-2 + \sqrt{3}i)^4 = (4 - 4\sqrt{3}i - 3)^2 = 1 - 8\sqrt{3}i - 48 = -47 - 8\sqrt{3}i$  實部為 -47, 虛部為  $-8\sqrt{3}$

4. 若  $(2-i)x^2 - 3(1-i)x - 2(1+i) = 0$  有實數解, 求另一虛根為\_\_\_\_\_ .

**解答**  $-\frac{1}{5} - \frac{3}{5}i$

**解析** 設方程式之實根為  $\alpha$ , 則  $(2-i)\alpha^2 - 3(1-i)\alpha - 2(1+i) = 0$

$\Rightarrow (2\alpha^2 - 3\alpha - 2) + (-\alpha^2 + 3\alpha - 2)i = 0$

$\Rightarrow \begin{cases} 2\alpha^2 - 3\alpha - 2 = 0 \\ \alpha^2 - 3\alpha + 2 = 0 \end{cases} \Rightarrow \begin{cases} (2\alpha+1)(\alpha-2) = 0 \\ (\alpha-1)(\alpha-2) = 0 \end{cases} \Rightarrow \begin{cases} \alpha = -\frac{1}{2} \text{ 或 } 2 \\ \alpha = 1 \text{ 或 } 2 \end{cases} \therefore \alpha = 2$

設另一根為  $\beta$ , 則  $2 + \beta = \frac{3(1-i)}{2-i} = \frac{3}{5}(3-i) \Rightarrow \beta = \frac{3}{5}(3-i) - 2 = \frac{-1-3i}{5}$

5.  $a, b \in \mathbb{R}$ ,  $\frac{2-3i}{a+bi} + \frac{1}{3+4i} = \frac{1}{3-4i}$ , 則  $(a, b) =$ \_\_\_\_\_ .

**解答**  $(-\frac{75}{8}, -\frac{25}{4})$

**解析**  $\frac{2-3i}{a+bi} + \frac{1}{3+4i} = \frac{1}{3-4i} \Rightarrow \frac{2-3i}{a+bi} = \frac{1}{3-4i} - \frac{1}{3+4i} = \frac{(3+4i) - (3-4i)}{3^2 + 4^2}$

$$\Rightarrow \frac{2-3i}{a+bi} = \frac{8i}{25} \Rightarrow \frac{a+bi}{2-3i} = \frac{25}{8i} \Rightarrow a+bi = \frac{25}{8i}(2-3i) = \frac{75}{8} - \frac{25}{4}i$$

6. 設  $a \in \mathbf{R}$ , 若二次方程式  $x^2 - ax - a + 8 = 0$  有相等實根, 則  $a$  為\_\_\_\_\_.

**解答** 4 或 -8

**解析**  $a \in \mathbf{R}$ ,  $x^2 - ax - a + 8 = 0$  有相等實根, 則  $D = (-a)^2 - 4(-a + 8) = 0 \Rightarrow a^2 + 4a - 32 = 0$   
 $\Rightarrow (a-4)(a+8) = 0 \Rightarrow a = 4$  或  $-8$

7. 設  $-2-i$  是實係數方程式  $ax^3 - 11x + b = 0$  的一根, 則  $(a, b) =$ \_\_\_\_\_.

**解答** (1, -20)

**解析**  $\because -2-i$  是實係數方程式  $ax^3 - 11x + b = 0$  之一根  $\Rightarrow -2+i$  亦為其根

$$[x - (-2-i)][x - (-2+i)]$$

$$= x^2 - (-2+i)x - (-2-i)x + 4+1 = x^2 + 4x + 5 \mid ax^3 - 11x + b$$

$$\Rightarrow \begin{array}{r} 1+4+5 \quad a-4a \\ a+0-11 \quad 0-11 \quad +b \\ a+4a+5a \\ -4a-(11+5a)+b \\ -4a \quad -16a-20a \\ \hline (-11+11a) + (b+20a) \end{array}$$

$$\Rightarrow \begin{cases} -11+11a=0 \cdots \cdots \textcircled{1} \\ b+20a=0 \cdots \cdots \textcircled{2} \end{cases} \Rightarrow a=1, b=-20$$

8. 設  $\alpha, \beta$  為  $2x^2 - 3x + 4 = 0$  的兩根, 則(1)  $\frac{\beta}{\alpha} + \frac{\alpha}{\beta} =$ \_\_\_\_\_. (2) 以  $\frac{\beta}{\alpha}, \frac{\alpha}{\beta}$  為二根的方程式為\_\_\_\_\_.

**解答** (1)  $-\frac{7}{8}$ ; (2)  $x^2 + \frac{7}{8}x + 1 = 0$

**解析**  $\alpha, \beta$  為  $2x^2 - 3x + 4 = 0$  的兩根  $\Rightarrow \begin{cases} \alpha + \beta = -\frac{-3}{2} = \frac{3}{2} \\ \alpha\beta = \frac{4}{2} = 2 \end{cases}$

$$(1) \frac{\beta}{\alpha} + \frac{\alpha}{\beta} = \frac{\beta^2 + \alpha^2}{\alpha\beta} = \frac{(\alpha + \beta)^2 - 2\alpha\beta}{\alpha\beta} = \frac{(\frac{3}{2})^2 - 2 \times 2}{2} = -\frac{7}{8}$$

$$(2) \text{ 以 } \frac{\beta}{\alpha}, \frac{\alpha}{\beta} \text{ 為二根的方程式為 } x^2 - (\frac{\beta}{\alpha} + \frac{\alpha}{\beta})x + (\frac{\beta}{\alpha} \cdot \frac{\alpha}{\beta}) = 0 \Rightarrow x^2 + \frac{7}{8}x + 1 = 0$$

9. 設  $m, k \in \mathbf{Q}$ ,  $m \neq 0$ , 且方程式  $2mx^2 - 3mx + 2x + m + k = 0$  之根為有理數, 則有理數  $k =$ \_\_\_\_\_.

**解答** -1 或 -2

**解析**  $\because$  方程式  $2mx^2 - 3mx + 2x + m + k = 0$  之根為有理數

$$\therefore (-3m+2)^2 - 4 \cdot 2m(m+k) = 9m^2 - 12m + 4 - 8m^2 - 8mk = m^2 - 2(6+4k)m + 4 \text{ 為完全平方式}$$

$$\Rightarrow (6+4k)^2 - 4 = 0 \Rightarrow (6+4k+2)(6+4k-2) = 0$$

$$\Rightarrow (k+2)(k+1) = 0 \therefore k = -1 \text{ 或 } k = -2$$

10.(1) 設  $\omega = \frac{-1+\sqrt{3}i}{2}$ , 則  $(3+\omega)(3+\omega^2)(3+\omega^3)(3+\omega^4)(3+\omega^5) =$ \_\_\_\_\_.

(2) 設  $\Omega = \frac{1+\sqrt{3}i}{2}$ , 則  $\Omega^{2012} =$  \_\_\_\_\_ .

**解答** (1)196;(2)  $\frac{-1+\sqrt{3}i}{2}$

**解析** (1) 若  $\omega = \frac{-1+\sqrt{3}i}{2}$ , 則  $\omega^3 = 1$  且  $1 + \omega + \omega^2 = 0$

$$\begin{aligned} & (3 + \omega)(3 + \omega^2)(3 + \omega^3)(3 + \omega^4)(3 + \omega^5) \\ &= (3 + \omega)(3 + \omega^2)(3 + 1)(3 + \omega)(3 + \omega^2) = 4 [(3 + \omega)(3 + \omega^2)]^2 \\ &= 4 [9 + 3(\omega + \omega^2) + \omega^3]^2 = 4 [9 + 3 \times (-1) + 1]^2 = 4 \times 49 = 196 \end{aligned}$$

$$(2) \Omega = \frac{1+\sqrt{3}i}{2} \Rightarrow (2\Omega - 1) = \sqrt{3}i \text{ 平方化簡} \Rightarrow \Omega^2 - \Omega + 1 = 0$$

$$\therefore (\Omega + 1)(\Omega^2 - \Omega + 1) = 0 \Rightarrow \Omega^3 + 1 = 0, \text{ 即 } \Omega^3 = -1$$

$$\Omega^{2012} = (\Omega^3)^{670} \cdot \Omega^2 = (-1)^{670} \cdot \Omega^2 = \Omega^2 = \frac{-1+\sqrt{3}i}{2}$$

11. 設  $\omega = \frac{-1+\sqrt{3}i}{2}$ , 則化簡  $(1 + \omega)^6 + (1 + \omega^2)^6 + (\omega + \omega^2)^6$  之值為 \_\_\_\_\_ .

**解答** 3

**解析**  $\because \omega = \frac{-1+\sqrt{3}i}{2} \therefore \omega^3 = 1, \omega^2 + \omega + 1 = 0$

$$\therefore (1 + \omega)^6 + (1 + \omega^2)^6 + (\omega + \omega^2)^6 = (-\omega^2)^6 + (-\omega)^6 + (-1)^6 = \omega^{12} + \omega^6 + 1 = (\omega^3)^4 + (\omega^3)^2 + 1 = 3$$

12. 設  $x, y$  是實數, 若  $(1+i)(x+2y) - (3-2i)(x-y) = 8+3i$ , 求 (1)  $x =$  \_\_\_\_\_ . (2)  $y =$  \_\_\_\_\_ .

**解答** (1)  $x = 1$ ; (2)  $y = 2$

**解析** 左式  $= x + 2y + xi + 2yi - (3x - 3y - 2xi + 2yi) = (-2x + 5y) + (3x)i = 8 + 3i$

$$\therefore \begin{cases} -2x + 5y = 8 \\ 3x = 3 \end{cases} \Rightarrow \begin{cases} x = 1 \\ y = 2 \end{cases}$$

13. 若  $\alpha, \beta$  為方程式  $x^2 + 7x + 9 = 0$  之兩根, 求 (1)  $(\sqrt{\alpha} - \sqrt{\beta})^2 =$  \_\_\_\_ . (2)  $(\alpha^2 + 10\alpha + 1)(\beta^2 + 10\beta + 1) =$  \_\_\_\_ .

**解答** (1) -1; (2) 313

**解析**  $\begin{cases} \alpha + \beta = -7 < 0 \\ \alpha\beta = 9 > 0 \end{cases}$  且  $D = 7^2 - 4 \times 1 \times 9 > 0 \Rightarrow \alpha < 0$  且  $\beta < 0$

$$(1) (\sqrt{\alpha} - \sqrt{\beta})^2 = (\sqrt{\alpha})^2 - 2\sqrt{\alpha}\sqrt{\beta} + (\sqrt{\beta})^2 = \alpha + 2\sqrt{\alpha\beta} + \beta = -7 + 2\sqrt{9} = -1$$

$$(2) \because \alpha, \beta \text{ 為 } x^2 + 7x + 9 = 0 \text{ 之兩根} \Rightarrow \begin{cases} \alpha^2 + 7\alpha + 9 = 0 \\ \beta^2 + 7\beta + 9 = 0 \end{cases} \Rightarrow \begin{cases} \alpha^2 = -7\alpha - 9 \\ \beta^2 = -7\beta - 9 \end{cases}$$

$$\begin{aligned} \therefore (\alpha^2 + 10\alpha + 1)(\beta^2 + 10\beta + 1) &= (-7\alpha - 9 + 10\alpha + 1)(-7\beta - 9 + 10\beta + 1) \\ &= (3\alpha - 8)(3\beta - 8) = 9\alpha\beta - 24(\alpha + \beta) + 64 = 9 \times 9 - 24 \times (-7) + 64 = 313 \end{aligned}$$

14.  $k$  為整數, 且  $x^4 - x^3 + kx^2 - 2kx - 2 = 0$  有有理根, 求  $k =$  \_\_\_\_\_ .

**解答** 0, -2

**解析** 所有可能的有理根為  $\pm 1, \pm 2$

- (1)若 1 為其根  $\Rightarrow 1-1+k-2k-2=0 \Rightarrow k=-2$   
 (2)若 -1 為其根  $\Rightarrow 1+1+k+2k-2=0 \Rightarrow k=0$   
 (3)若 2 為其根  $\Rightarrow 16-8+4k-4k-2=0 \Rightarrow 6=0, k$  無解  
 (4)若 -2 為其根  $\Rightarrow 16+8+4k+4k-2=0 \Rightarrow k=-\frac{22}{8}$  (不合)

故  $k=0, -2$

15.若  $a \in \mathbf{R}, a \neq 0$ , 設方程式  $f(x) = ax^2 - (a-1)x - 6 = 0$  在區間  $(-2, -1)$  與  $(1, 2)$  中各有一個實根, 則  $a$  之範圍為\_\_\_\_\_.

**解答**  $2 < a < \frac{7}{2}$

**解析**  $f(-2) = 4a + 2a - 2 - 6 = 6a - 8, f(-1) = a + a - 1 - 6 = 2a - 7$

$\therefore$  在區間  $(-2, -1)$  之間有一根

$$\therefore f(-2)f(-1) < 0 \Rightarrow (6a-8)(2a-7) < 0 \Rightarrow (3a-4)(2a-7) < 0 \Rightarrow \frac{4}{3} < a < \frac{7}{2} \dots\dots \textcircled{1}$$

$$\text{又 } f(1) = a - a + 1 - 6 = -5, f(2) = 4a - 2a + 2 - 6 = 2a - 4$$

$\therefore$  在區間  $(1, 2)$  之間有一根

$$\therefore f(1)f(2) < 0 \Rightarrow -5(2a-4) < 0 \Rightarrow a-2 > 0 \Rightarrow a > 2 \dots\dots \textcircled{2}$$

由  $\textcircled{1}, \textcircled{2}$  得  $2 < a < \frac{7}{2}$ , 故實數  $a$  的範圍為  $2 < a < \frac{7}{2}$

16.設一多項式  $f(x) = x^5 - x^2 + 2x - 3$ , 若介於兩連續整數  $n$  與  $n+1$  之間恰有一實根, 則  $n =$ \_\_\_\_\_.

**解答** 1

**解析**

	$f(x)$	$x$
$1+0+0-1+2$	$-3$	$0$
$1+1+1+0+2$	$-1$	$1$
$1+2+4+7+16$	$+29$	$2$

$\left. \begin{array}{l} \\ \\ \end{array} \right\} f(1) \cdot f(2) < 0$   
則  $n=1$

17.設 1 為  $x^4 - 5x^3 + ax^2 + bx + c = 0$  之三重根, 則(1)另一根為\_\_\_\_\_. (2) $b+c =$ \_\_\_\_\_.

**解答** (1)2;(2)-5

**解析** 已知 1 為  $x^4 - 5x^3 + ax^2 + bx + c = 0$  之三重根, 設另一根為  $\alpha$

由根與係數關係, 四根和為  $1+1+1+\alpha=5 \Rightarrow \alpha=2$

$$\therefore x^4 - 5x^3 + ax^2 + bx + c = (x-1)^3(x-2) = (x^3 - 3x^2 + 3x - 1)(x-2) = x^4 - 5x^3 + 9x^2 - 7x + 2$$

$$\therefore b+c = -7+2 = -5$$

18.設  $f(x) = a_3x^3 + a_2x^2 + a_1x + a_0$  為實係數三次多項式, 若  $f(3+2i) = -5+4i$ , 則  $f(3-2i) =$ \_\_\_\_\_.

**解答**  $-5-4i$

**解析**  $f(x)$  為實係數多項式,  $z$  為複數, 則  $f(\bar{z}) = \overline{f(z)}$

$$\text{故 } f(3-2i) = \overline{f(3+2i)} = \overline{-5+4i} = -5-4i$$

19.若一有理係數方程式有  $2-\sqrt{3}$  及  $1-i$  兩根, 則此最低次方程式為\_\_\_\_\_.

**解答**  $x^4 - 6x^3 + 11x^2 - 10x + 2 = 0$

**解析** 有理數係數方程式有  $2-\sqrt{3}$  及  $1-i$  兩根, 必有另兩根  $2+\sqrt{3}$  及  $1+i$

∴ 此最低次方程式為  $[x - (2 - \sqrt{3})][x - (2 + \sqrt{3})][x - (1 - i)][x - (1 + i)]$   
 即  $[(x - 2)^2 - (\sqrt{3})^2][(x - 1)^2 - (i)^2] = (x^2 - 4x + 1)(x^2 - 2x + 2) = x^4 - 6x^3 + 11x^2 - 10x + 2$

20. 方程式  $6x^4 + 5x^3 + 9x^2 - 4x - 4 = 0$  之有理根為\_\_\_\_\_。

**解答**  $-\frac{1}{2}, \frac{2}{3}$

**解析** 設  $ax - b$  為其整係數一次因式,  $(a, b) = 1$ , 則  $a | 6, b | -4$

∴  $\frac{b}{a}$  之可能值為  $\pm 1, \pm 2, \pm 4, \pm \frac{1}{2}, \pm \frac{1}{3}, \pm \frac{2}{3}, \pm \frac{4}{3}, \pm \frac{1}{6}$ , 利用綜合除法

$$\begin{array}{r|rrrrrr} & 6 & 5 & 9 & -4 & -4 & \\ & & -3 & -1 & -4 & +4 & \\ \hline 2 & 6 & 2 & 8 & -8 & 0 & \\ & & 3 & 1 & 4 & -4 & \\ & & & +2 & +2 & +4 & \\ \hline 3 & 3 & 3 & 6 & 0 & & \\ & & 1 & 1 & 2 & & \end{array}$$

∴ 有理根為  $-\frac{1}{2}, \frac{2}{3}$

21. 設  $\alpha, \beta, \gamma$  為  $x^3 - 6x^2 + 11x - 7 = 0$  的三根, 則

(1)  $\alpha^3 + \beta^3 + \gamma^3 =$  \_\_\_\_\_ . (2)  $(\alpha + \beta - 7)(\beta + \gamma - 7)(\gamma + \alpha - 7) =$  \_\_\_\_\_ .

**解答** (1) 39; (2) -25

**解析** (1) 已知  $\begin{cases} \alpha + \beta + \gamma = 6 \\ \alpha\beta + \beta\gamma + \gamma\alpha = 11 \\ \alpha\beta\gamma = 7 \end{cases}$

$$\begin{aligned} \alpha^3 + \beta^3 + \gamma^3 - 3\alpha\beta\gamma &= (\alpha + \beta + \gamma)(\alpha^2 + \beta^2 + \gamma^2 - \alpha\beta - \beta\gamma - \gamma\alpha) \\ &= (\alpha + \beta + \gamma)[(\alpha + \beta + \gamma)^2 - 3(\alpha\beta + \beta\gamma + \gamma\alpha)] \\ \Rightarrow \alpha^3 + \beta^3 + \gamma^3 - 21 &= 6 \times (36 - 33), \text{ 得 } \alpha^3 + \beta^3 + \gamma^3 = 39 \end{aligned}$$

(2) ∵  $\alpha + \beta + \gamma = 6$  ∴  $\alpha + \beta = 6 - \gamma, \beta + \gamma = 6 - \alpha, \gamma + \alpha = 6 - \beta$

$$\text{則 } (\alpha + \beta - 7)(\beta + \gamma - 7)(\gamma + \alpha - 7) = (6 - \gamma - 7)(6 - \alpha - 7)(6 - \beta - 7)$$

$$= -(\alpha + 1)(\beta + 1)(\gamma + 1) = -[\alpha\beta\gamma + (\alpha\beta + \beta\gamma + \gamma\alpha) + (\alpha + \beta + \gamma) + 1] = -(7 + 11 + 6 + 1) = -25$$

22. 設  $x^2 + (m^2 - m - 6)x - m^2 - 5m + 7 = 0$

(1) 若兩根互為倒數, 則  $m =$  \_\_\_\_\_ . (2) 若兩根互為相反數, 則  $m =$  \_\_\_\_\_ .

**解答** (1) -6 或 1; (2) 3 或 -2

**解析** (1) 兩根之積為 1  $\Rightarrow -m^2 - 5m + 7 = 1 \Rightarrow m^2 + 5m - 6 = 0 \Rightarrow m = -6$  或  $1$   
 (2) 兩根之和為 0  $\Rightarrow m^2 - m - 6 = 0 \Rightarrow m = 3$  或  $-2$

23.  $a, b \in \mathbf{R}, \omega = \frac{-1 + \sqrt{3}i}{2}$ , 若  $\frac{1}{3 - \omega} = a + b\omega$ , 則數對  $(a, b) =$  \_\_\_\_\_ .

**解答**  $(\frac{4}{13}, \frac{1}{13})$

**解析** ∵  $\omega = \frac{-1 + \sqrt{3}i}{2} \Rightarrow \omega^3 = 1$  且  $1 + \omega + \omega^2 = 0$

$$\text{又 } \frac{1}{3-\omega} = a + b\omega \Rightarrow 1 = (3-\omega)(a+b\omega) = 3a + 3b\omega - a\omega - b\omega^2$$

$$\Rightarrow 1 = 3a + 3b\omega - a\omega - b(-1-\omega) = (3a+b) + (-a+4b)\omega \Rightarrow \begin{cases} 3a+b=1 \\ -a+4b=0 \end{cases} \Rightarrow a = \frac{4}{13}, b = \frac{1}{13}$$

24. 若  $z^2 = 7 - 24i$ , 則  $z =$  \_\_\_\_\_ .

**解答**  $\pm(4-3i)$

**解析** 令  $(x+yi)^2 = 7-24i$ ,  $x, y \in \mathbf{R} \Rightarrow (x^2-y^2) + 2xyi = 7-24i$

$$\Rightarrow \begin{cases} x^2 - y^2 = 7 \cdots \cdots \textcircled{1} \\ 2xy = -24 \cdots \cdots \textcircled{2} \end{cases}, \sqrt{\textcircled{1}^2 + \textcircled{2}^2} \Rightarrow x^2 + y^2 = 25 \cdots \cdots \textcircled{3}$$

$$\textcircled{1} + \textcircled{3} \quad \therefore 2x^2 = 32 \quad \therefore x = \pm 4 \text{ 代入 } \textcircled{2} \therefore y = \mp 3 \Rightarrow x + yi = \pm(4-3i), \text{ 即 } z = \pm(4-3i)$$

25. 若  $a, b$  均為整數且方程式  $x^2 - ax + 91 = 0$  與  $x^2 - bx + 143 = 0$  有一共同的質數根, 則(1)  $a =$  \_\_\_\_\_, (2)  $b =$  \_\_\_\_\_.

**解答** (1)20; (2)24

**解析**  $91 = 7 \times 13, 143 = 11 \times 13, \therefore$  共同的質數根為 13, 故  $a = 7 + 13 = 20, b = 11 + 13 = 24$ .

26. 設兩實係數多項式  $f(x) = x^4 - 3x^3 + 6x^2 + ax - 50$  與  $g(x) = x^2 + ax + b$ , 且  $g(x)$  滿足  $g(1+3i) = 0$ , 求  $f(1-3i) =$  \_\_\_\_\_ .

**解答**  $6+12i$

**解析**  $x = 1+3i \Rightarrow (x-1)^2 = (3i)^2 \Rightarrow x^2 - 2x + 10 = 0 \Rightarrow a = -2,$   
 $\therefore f(x) = x^4 - 3x^3 + 6x^2 - 2x - 50 = (x^2 - 2x + 10)(x^2 - x - 6) - 4x + 10 \Rightarrow$   
 $f(1-3i) = -4(1-3i) + 10 = 6 + 12i$ .

27. 方程式  $f(x) = x^3 + x - 3 = 0$  在 1 與 2 之間有一實根, 在誤差要小於  $\frac{1}{10}$  的情況之下, 求其近似值 = \_\_\_\_\_.

**解答**  $\frac{19}{16}$

**解析**  $f(1) = -1 < 0, f(2) = 7 > 0 \Rightarrow$  根在 1~2 之間

取 1 與 2 中點  $\frac{3}{2}, f(\frac{3}{2}) = \frac{15}{8} > 0 \Rightarrow$  根在  $1 \sim \frac{3}{2}$  之間

取 1 與  $\frac{3}{2}$  中點  $\frac{5}{4}, f(\frac{5}{4}) = \frac{13}{64} > 0 \Rightarrow$  根在  $1 \sim \frac{5}{4}$  之間

取 1 與  $\frac{5}{4}$  中點  $\frac{9}{8}, f(\frac{9}{8}) = -\frac{231}{512} < 0 \Rightarrow$  根在  $\frac{9}{8} \sim \frac{5}{4}$  之間 (區間長度  $\frac{1}{8}$ )

故取  $\frac{9}{8}$  與  $\frac{5}{4}$  中點  $\frac{19}{16}$  做為根的近似值, 誤差就會小於  $\frac{1}{16}$ , 當然也就小於  $\frac{1}{10}$

$$\therefore x \doteq \frac{19}{16}$$

28. 設  $x, y$  為兩個非零複數, 滿足  $x^2 + xy + y^2 = 0$ , 則  $(\frac{x}{x+y})^{2008} + (\frac{y}{x+y})^{2008}$  之值為 \_\_\_\_\_ .

**解答**  $-1$

**解析**  $\therefore x^2 + xy + y^2 = 0, \therefore (\frac{y}{x})^2 + (\frac{y}{x}) + 1 = 0 \Rightarrow \frac{y}{x} = \frac{-1 \pm \sqrt{3}i}{2} = \omega$  且  $\omega^3 = 1$  且  $\omega^2 + \omega + 1 = 0$

$$\begin{aligned}\text{求值式} &= \left(\frac{1}{1+\frac{y}{x}}\right)^{2008} + \left(\frac{\frac{y}{x}}{1+\frac{y}{x}}\right)^{2008} = \left(\frac{1}{1+\omega}\right)^{2008} + \left(\frac{\omega}{1+\omega}\right)^{2008} \\ &= \left(\frac{1}{-\omega^2}\right)^{2008} + \left(\frac{\omega}{-\omega^2}\right)^{2008} = \frac{1}{\omega^{4016}} + \frac{1}{\omega^{2008}} = \frac{1}{\omega^2} + \frac{1}{\omega} = \frac{1+\omega}{\omega^2} = \frac{-\omega^2}{\omega^2} = -1\end{aligned}$$