

# 第 4 回 解答

## 一、填充題

■  $S_{XX}, S_{YY}, S_{XY}, \frac{S_{XY}}{S_{XX}}, \bar{y} - a\bar{x}$

## 二、多重選擇題

1. (1)(3)(5)    2. (1)(2)(4)    3. (2)(5)

## 三、填充題

1. (1) 1600    (2)  $\frac{2}{5}x + 37$     2. (1)  $\frac{685}{16}x + \frac{2405}{8}$     (2) 986

-----《解析》-----

## 二、多重選擇題

1. (1)  $(\bar{x}, \bar{y}) = (\frac{1}{5}(-3-1+2+3+4), \frac{1}{5}(-2+1+1+5+5)) = (1, 2)$

(2)  $S_{XY} = (-4)(-4) + (-2)(-1) + 1(-1) + 2 \cdot 3 + 3 \cdot 3 = 32$

(3)  $S_{XX} = (-4)^2 + (-2)^2 + 1^2 + 2^2 + 3^2 = 34$

(4)  $S_{YY} = (-4)^2 + (-1)^2 + (-1)^2 + 3^2 + 3^2 = 36$

(5)  $r_{XY} = \frac{32}{\sqrt{34 \cdot 36}} \approx 0.91$

故選(1)(3)(5)

2. (1)  $\bar{x} = \frac{1}{30} \sum_{i=1}^{30} x_i = 5, \bar{y} = \frac{1}{30} \sum_{i=1}^{30} y_i = 8 \therefore \bar{x} + \bar{y} = 13$

(2)  $S_{XX} = \sum_{i=1}^{30} x_i^2 - 30\bar{x}^2 = 825 - 30 \times 5^2 = 75$

(3)  $S_{YY} = \sum_{i=1}^{30} y_i^2 - 30\bar{y}^2 = 2067 - 30 \times 8^2 = 147$

(4)  $S_{XY} = \sum_{i=1}^{30} x_i y_i - 30\bar{x}\bar{y} = 1290 - 30 \times 5 \times 8 = 90$

(5)  $r_{XY} = \frac{S_{XY}}{\sqrt{S_{XX}S_{YY}}} = \frac{90}{\sqrt{75 \times 147}} = \frac{90}{105} = \frac{6}{7} \approx 0.86$

故選(1)(2)(4)

3. (1)  $S_{YY} = \sum_{j=1}^n (y_j - \bar{y})^2 = \sum_{j=1}^n [(-3x_j + 2) - (-3\bar{x} + 2)]^2 = \sum_{j=1}^n [(-3)(x_j - \bar{x})]^2 = 9 \sum_{j=1}^n (x_j - \bar{x})^2 = 9S_{XX}$

(2)  $S_{ZZ} = \sum_{j=1}^n (z_j - \bar{z})^2 = \sum_{j=1}^n [(2x_j - 1) - (2\bar{x} - 1)]^2 = \sum_{j=1}^n [2(x_j - \bar{x})]^2 = 4 \sum_{j=1}^n (x_j - \bar{x})^2 = 4S_{XX}$

(3)  $S_{XY} = \sum_{j=1}^n (x_j - \bar{x})(y_j - \bar{y}) = \sum_{j=1}^n (x_j - \bar{x})(-3)(x_j - \bar{x}) = -3 \sum_{j=1}^n (x_j - \bar{x})^2 = -3S_{XX}$

(4)  $r_{XY} = \frac{S_{XY}}{\sqrt{S_{XX}S_{YY}}} = \frac{-3S_{XX}}{\sqrt{S_{XX} \cdot 9S_{XX}}} = \frac{-3S_{XX}}{3S_{XX}} = -1$

(5)  $S_{XZ} = \sum_{j=1}^n (x_j - \bar{x})(z_j - \bar{z}) = \sum_{j=1}^n (x_j - \bar{x})(2)(x_j - \bar{x}) = 2 \sum_{j=1}^n (x_j - \bar{x})^2 = 2S_{XX}$

$\therefore r_{XZ} = \frac{S_{XZ}}{\sqrt{S_{XX}S_{ZZ}}} = \frac{2S_{XX}}{\sqrt{S_{XX} \cdot 4S_{XX}}} = \frac{2S_{XX}}{2S_{XX}} = 1$

(4)(5)兩選項也可以看成在線性轉換過程中，斜率為正（負），其相關係數即為 1（-1）

故選(2)(5)

### 三、填充題

$$1. (1) S_x = \sqrt{\frac{1}{n} \sum_{i=1}^n (x_i - \bar{x})^2} = \sqrt{\frac{1}{n} S_{xx}}$$

$$\Rightarrow S_{xx} = nS_x^2 = 40 \times 10^2 = 4000, \text{ 同理 } S_{yy} = 40 \times 5^2 = 1000$$

$$\text{而 } r_{xy} = \frac{S_{xy}}{\sqrt{S_{xx}S_{yy}}} \Rightarrow S_{xy} = 0.8 \times \sqrt{4000 \times 1000} = 0.8 \times 2000 = 1600$$

$$(2) \text{ 設迴歸直線為 } y = ax + b, \text{ 則 } a = \frac{S_{xy}}{S_{xx}} = \frac{1600}{4000} = \frac{2}{5}, b = \bar{y} - a\bar{x} = 65 - \frac{2}{5} \times 70 = 37$$

$$\therefore \text{迴歸直線方程式為 } y = \frac{2}{5}x + 37$$

$$2. (1) \bar{x} = 14, \bar{y} = 900$$

$$\therefore S_{xy} = (-4)(-150) + (-3)(-190) + (-2)(-50) + 1 \cdot (-30) + 5 \cdot 120 + 3 \cdot 300 = 2740$$

$$S_{xx} = (-4)^2 + (-3)^2 + (-2)^2 + 1^2 + 5^2 + 3^2 = 64$$

$$\therefore a = \frac{2740}{64} = \frac{685}{16}, b = 900 - \frac{685}{16} \cdot 14 = \frac{2405}{8}$$

$$\therefore \text{迴歸直線方程式為 } y = \frac{685}{16}x + \frac{2405}{8}$$

$$(2) x = 16 \text{ 代入得 } y = \frac{685}{16} \cdot 16 + \frac{2405}{8} \approx 986 \text{ (萬美元)}$$

### 四、計算題

$$\blacksquare (1) \bar{x} = 20, \bar{y} = 40 \quad \therefore S_{xx} = (-10)^2 + (-5)^2 + 0 + 5^2 + 10^2 = 250$$

$$\Rightarrow S_x = \sqrt{\frac{1}{4} \times 250} = \frac{5\sqrt{10}}{2}$$

$$(2) S_{xy} = (-10)(-14) + (-5)(-8) + 0 \cdot (-1) + 5 \cdot 7 + 10 \cdot 16 = 375$$

$$(3) \text{ 設迴歸直線 } y = ax + b, \text{ 則 } a = \frac{375}{250} = \frac{3}{2}, b = \bar{y} - a\bar{x} = 40 - \frac{3}{2} \cdot 20 = 10$$

$$\therefore \text{迴歸直線方程式為 } y = \frac{3}{2}x + 10$$

$$(4) x = 28 \text{ 代入得 } y = \frac{3}{2} \times 28 + 10 = 52$$