

第 1 回 解答

一、多重選擇題

1. (1)(2)(5) 2. (1)(3)(4)

二、填充題

1. $\frac{3}{8}$ 2. 7 3. $\frac{19}{36}$ 4. 14 5. $\frac{6}{11}$ 6. $\frac{13}{30}$

-----《解析》-----

一、多重選擇題

1. 令 A 表示取得產品合格的事件, A' 即為不合格的事件

$$\begin{aligned} P(A') &= P(\text{甲} \cap A') + P(\text{乙} \cap A') + P(\text{丙} \cap A') \\ &= P(\text{甲})P(A'|\text{甲}) + P(\text{乙})P(A'|\text{乙}) + P(\text{丙})P(A'|\text{丙}) \\ &= 45\% \cdot 8\% + 25\% \cdot 12\% + 30\% \cdot 10\% = 0.096 = 9.6\% \end{aligned}$$

$$\Rightarrow P(A) = 1 - P(A') = 90.4\%$$

$$P(\text{甲}|A') = \frac{P(\text{甲} \cap A')}{P(A')} = \frac{45\% \cdot 8\%}{9.6\%} = \frac{36}{96} = \frac{3}{8},$$

$$\text{同理 } P(\text{乙}|A') = \frac{25\% \cdot 12\%}{9.6\%} = \frac{30}{96} = \frac{5}{16}$$

$$P(\text{丙}|A') = \frac{30\% \cdot 10\%}{9.6\%} = \frac{30}{96} = \frac{5}{16}, \quad P(\text{甲}|A) = \frac{45\% \cdot 92\%}{90.4\%} = \frac{414}{904} = \frac{207}{452}$$

$$P(\text{乙}|A) = \frac{25\% \cdot 88\%}{90.4\%} = \frac{220}{904} = \frac{55}{226}, \quad P(\text{丙}|A) = \frac{30\% \cdot 90\%}{90.4\%} = \frac{270}{904} = \frac{135}{452}$$

故(1) $P(A) = 90.4\% > 90\%$, 為真

(2) $P(\text{甲}|A) > P(\text{丙}|A) > P(\text{乙}|A)$, 為真

(3) $P(\text{甲}|A') > P(\text{乙}|A') = P(\text{丙}|A')$

$$(4) P(\text{丙}|A) = \frac{135}{452}$$

$$(5) P(\text{丙}|A') = \frac{5}{16}, \text{ 為真}$$

選(1)(2)(5)

2. (1) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$$= \frac{1}{3} + \frac{2}{5} - \frac{1}{15} = \frac{10}{15} = \frac{2}{3}$$

$$(2) P(A' \cap B) = P(B) - P(A \cap B) = \frac{2}{5} - \frac{1}{15} = \frac{1}{3}$$

$$(3) P(A \cup B') = P((A' \cap B)') = P(S) - P(A' \cap B) = 1 - \frac{1}{3} = \frac{2}{3}$$

$$(4) P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{\frac{1}{15}}{\frac{2}{5}} = \frac{1}{6}$$

$$(5) P(B|A') = \frac{P(A' \cap B)}{P(A')} = \frac{\frac{1}{3}}{\frac{2}{3}} = \frac{1}{2}$$

故選(1)(3)(4)

二、填充題

$$1. \because P(A \cup B) = P(A) + P(B) - P(A \cap B) \Rightarrow \frac{2}{5} + P(B) - \frac{2}{5}P(B) = \frac{31}{40} \Rightarrow \frac{3}{5}P(B) = \frac{15}{40}$$

$$\therefore P(B) = \frac{5}{8}, \text{ 故 } P(A' \cap B) = P(A')P(B) = \frac{3}{5} \times \frac{5}{8} = \frac{3}{8}$$

2. 至少出現一次正面即全部去掉正面皆不出現
設最少要擲 n 次

$$\text{依題意 } 1 - \left(\frac{1}{2}\right)^n > \frac{99}{100} \Rightarrow \left(\frac{1}{2}\right)^n < \frac{1}{100}$$

解得 n 最小值為 7

3. $P(\text{紅}) = P(\text{甲})P(\text{紅}|\text{甲}) + P(\text{乙})P(\text{紅}|\text{乙}) + P(\text{丙})P(\text{紅}|\text{丙})$

$$= \frac{1}{3} \times \frac{1}{4} + \frac{1}{3} \times \frac{1}{3} + \frac{1}{3} \times 1 = \frac{19}{36}$$

4. 分別以 A, B, C 代表三種年齡層的族群, X 代表吸煙的族群

$$\therefore P(X) = P(A)P(X|A) + P(B)P(X|B) + P(C)P(X|C)$$

$$\Rightarrow 21\% = 40\% \cdot P(X|A) + 25\% \cdot 28\% + 35\% \cdot 24\%$$

$$\text{故所求 } P(X|A) = \frac{0.21 - 0.07 - 0.084}{0.4} = 0.14 = 14\%$$

5. 令 A : 至少出現兩次正面, B : 恰好出現兩次正面的事件

$$\therefore n(A) = \text{全} - (\text{四反}) - (\text{三反一正}) = 2^4 - 1 - \frac{4!}{3!1!} = 11$$

$$n(A \cap B) = n(B) = \frac{4!}{2!2!} = 6$$

$$\text{故所求 } P(B|A) = \frac{n(A \cap B)}{n(A)} = \frac{6}{11}$$

6. 騙人布說選中女生有兩種情形: 選到女生而且他說實話, 選到男生而且他說謊話

$$\therefore \text{所求機率為 } \frac{1}{6} \times \frac{3}{5} + \frac{5}{6} \times \frac{2}{5} = \frac{13}{30}$$

三、計算題

1. (1) A 事件為 $(1, 3, 6), (1, 4, 5), (2, 2, 6), (2, 3, 5), (2, 4, 4), (3, 3, 4)$
而前後三次出現的點數可互換

$$\therefore n(A) = 6 + 6 + 3 + 6 + 3 + 3 = 27$$

又 $A \cap B$ 事件為 $(1, 3, 6), (2, 3, 5), (3, 3, 4)$

$$\therefore n(A \cap B) = 6 + 6 + 3 = 15$$

$$\text{故 } P(B|A) = \frac{n(A \cap B)}{n(A)} = \frac{15}{27} = \frac{5}{9}$$

- (2) B 事件即全部去掉點數 3 皆不出現

$$\therefore n(B) = 6^3 - 5^3 = 91 \quad \therefore P(A|B) = \frac{n(A \cap B)}{n(B)} = \frac{15}{91}$$

2. (1) 令 A, B, C 分別表示佐助、鳴人、小櫻打中的事件, 目標被打中即全部情形去掉三人都沒打中
 \therefore 所求為 $1 - P(A' \cap B' \cap C') = 1 - P(A')P(B')P(C') = 1 - 0.2 \times 0.5 \times 0.3 = 0.97$

- (2) 令 T 表目標被打中一枚暗器的事件

$$\therefore P(T) = P(A \cap B' \cap C') + P(A' \cap B \cap C') + P(A' \cap B' \cap C)$$

$$= 0.8 \times 0.5 \times 0.3 + 0.2 \times 0.5 \times 0.3 + 0.2 \times 0.5 \times 0.7 = 0.22$$

$$\text{所求即 } P(C|T) = \frac{P(A' \cap B' \cap C)}{P(T)} = \frac{0.07}{0.22} = \frac{7}{22}$$