高雄市明誠中學 高三數學平時測驗 日期：100.12.08					
範圍	第 7 回三角函數(3)	班級	三年 班	姓名	
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一、填充題(每題 10 分)

1、 $f(x)=\cos^2 x-3\sin x+1$，x 為實數，試求 $f(x)$ 之最大值為________，最小值為________.

答案： 4;−2

解析： $f(x)=(1-\sin^2 x)-3\sin x+1=-[\sin^2 x+3\sin x+(\frac{3}{2})^2]+\frac{17}{4}=-(\sin x+\frac{3}{2})^2+\frac{17}{4}$,

$\because -1\le \sin x\le 1$,

$\sin x=1, f(x)=-1-3+2=-2$ 為最小值，

$\sin x=-1, f(x)=-1+3+2=4$ 為最大值.

2、 當 $-\frac{\pi}{4}\le x\le \frac{\pi}{3}$ 時，試求 $f(x)=-\sec^2 x+2\tan x+1$ 的最大值為________.

答案：1

解析： $f(x)=-(1+\tan^2 x)+2\tan x+1$

$=-\tan^2 x+2\tan x$

$=-(\tan^2 x-2\tan x+1)+1$

$=-(\tan x-1)^2+1$

當 $\tan x=1, x=\frac{\pi}{4}$ 時， $f(x)$ 有最大值 1.

3、 $f(x)=3\cos x-4\sin x+1$，且 x 為實數，試求 $f(x)$ 之最大值為________，最小值為________.

答案： 6;−4

解析： $f(x)=5(\frac{3}{5}\cos x-\frac{4}{5}\sin x)+1=5(\sin\phi\cos x-\cos\phi\sin x)+1=5\sin(\phi-x)+1$,

$\because -1\le \sin(\phi-x)\le 1$,

$\therefore -5\le 5\sin(\phi-x)\le 5\Rightarrow -4\le 5\sin(\phi-x)+1\le 6$， 最大值 $=6$ ，最小值 $=-4$.

4、 $f(x)=2\sin(x-\frac{\pi}{6})+2\cos x-1, -\frac{\pi}{6}\le x\le \frac{5\pi}{6}$ ，則

(1)當 $x=$________時， $f(x)$ 之最大值為________；

(2)當 $x=$________時， $f(x)$ 之最小值為________.

答案： $\frac{\pi}{3};1;\frac{-\pi}{6},\frac{5\pi}{6};-1$

解析： $f(x)=2\sin(x-\frac{\pi}{6})+2\cos x-1, -\frac{\pi}{6}\le x\le \frac{5\pi}{6}$

$=2[\sin x\cos\frac{\pi}{6}-\cos x\sin\frac{\pi}{6}]+2\cos x-1$

$=2[\sin x\cdot\frac{\sqrt{3}}{2}-\frac{1}{2}\cos x]+2\cos x-1$

$=\sqrt{3}\sin x+\cos x-1$

$=2[\sin x\cdot\frac{\sqrt{3}}{2}+\cos x\cdot\frac{1}{2}]-1=2[\sin x\cos\frac{\pi}{6}+\cos x\sin\frac{\pi}{6}]-1=2\sin(x+\frac{\pi}{6})-1$,

$\because \frac{-\pi}{6}\le x\le\frac{5\pi}{6}\Rightarrow 0\le x+\frac{\pi}{6}\le\pi$,

$\Rightarrow 0\le\sin(x+\frac{\pi}{6})\le 1\Rightarrow 0\le 2\sin(x+\frac{\pi}{6})\le 2\Rightarrow -1\le 2\sin(x+\frac{\pi}{6})-1\le 1$,

當 $x+\frac{\pi}{6}=\frac{\pi}{2}\Rightarrow x=\frac{\pi}{3}$ ， $f(x)$ 有最大值 $=1$

當 $x+\frac{\pi}{6}=0$ 或 $x=-\frac{\pi}{6},\frac{5\pi}{6}$ ， $f(x)$ 有最小值 $=-1$.

5、 若 x 爲實數，則 $\frac{1+\sin x}{3+\cos x}$ 之最大值爲________，最小值爲________.

答案： $\frac{3}{4};0$

解析：設 $y=\frac{1+\sin x}{3+\cos x}\Rightarrow 1+\sin x=3y+y\cos x$

$\Rightarrow \sin x-y\cos x=3y-1$

$\Rightarrow \sqrt{1+y^2}(\sin x\cdot\frac{1}{\sqrt{1+y^2}}-\cos x\cdot\frac{y}{\sqrt{1+y^2}})=3y-1$

$\Rightarrow \sqrt{1+y^2}\sin(x-\theta)=3y-1$，其中 $\cos\theta=\frac{1}{\sqrt{1+y^2}}$, $\sin\theta=\frac{y}{\sqrt{1+y^2}}$

$\Rightarrow \sin(x-\theta)=\frac{3y-1}{\sqrt{1+y^2}}$

又 $-1\le\sin(x-\theta)\le 1\Rightarrow -1\le\frac{3y-1}{\sqrt{1+y^2}}\le 1$

$\Rightarrow |\frac{3y-1}{\sqrt{1+y^2}}|\le 1$

$\Rightarrow |3y-1|\le\sqrt{1+y^2}$

$\Rightarrow (3y-1)^2\le 1+y^2\Rightarrow 4y^2-3y\le 0$

$\Rightarrow y(4y-3)\le 0 \quad \Rightarrow 0\le y\le\frac{3}{4}$ ， 最大值 $=\frac{3}{4}$，最小值 $=0$.

6、 若 θ 在第二象限，試求： $\tan\theta+\cot\theta$ 之最大值爲________.

答案： -2

解析： $\because \theta$ 在第二象限 $\Rightarrow \tan\theta<0,\quad \cot\theta<0$

$\frac{(-\tan\theta)+(-\cot\theta)}{2}\ge\sqrt{(-\tan\theta)(-\cot\theta)}$ (算幾不等式)

$\Rightarrow -(\tan\theta+\cot\theta)\ge 2\cdot 1$

$\Rightarrow \tan\theta+\cot\theta\le -2$, $\therefore$ 最大值爲 -2.

7、 $f(x)=\sin^2 x+2\sin x\cos x+3\cos^2 x$，則 $f(x)$ 之最大值爲________，最小值爲________.

答案：$2+\sqrt{2};2-\sqrt{2}$

解析：$f(x)=\dfrac{1-\cos 2x}{2}+\sin 2x+3\cdot\dfrac{1+\cos 2x}{2}$

$$=\sin 2x+\cos 2x+2=\sqrt{2}(\frac{1}{\sqrt{2}}\sin 2x+\frac{1}{\sqrt{2}}\cos 2x)+2$$

$$=\sqrt{2}(\sin 2x\cos\frac{\pi}{4}+\cos 2x\sin\frac{\pi}{4})+2=\sqrt{2}\sin(2x+\frac{\pi}{4})+2,$$

$\because -1\le\sin(2x+\frac{\pi}{4})\le 1\Rightarrow -\sqrt{2}\le\sqrt{2}\sin(2x+\frac{\pi}{4})\le\sqrt{2}$

$\Rightarrow 2-\sqrt{2}\le\sqrt{2}\sin(2x+\frac{\pi}{4})+2\le 2+\sqrt{2}$,

最大值 $=2+\sqrt{2}$，最小值 $=2-\sqrt{2}$.

8、 $\sin(\theta+60°)+2\sin(\theta-60°)-\sqrt{3}\cos(120°-\theta)=$________.

答案：0

解析：$\because (\theta+60°)+(120°-\theta)=180°$,

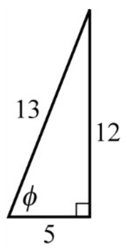
$\therefore \cos(120°-\theta)=\cos[180°-(\theta+60°)]=-\cos(\theta+60°)$,

原式 $=[\sin(\theta+60°)+\sqrt{3}\cos(\theta+60°)]+2\sin(\theta-60°)$

$$=2[\frac{1}{2}\sin(\theta+60°)+\frac{\sqrt{3}}{2}\cos(\theta+60°)]+2\sin(\theta-60°)$$
$$=2[\sin(\theta+60°)\cos 60°+\cos(\theta+60°)\sin 60°]+2\sin(\theta-60°)$$
$$=2\sin(\theta+60°+60°)-2\sin(60°-\theta)$$
$$=2\sin(\theta+120°)-2\sin(60°-\theta)$$
$$=2\sin(180°-(60°-\theta))-2\sin(60°-\theta)$$
$$=2\sin(60°-\theta)-2\sin(60°-\theta)=0.$$

9、 若 $f(x)=5\cos x-12\sin x$，且當 $x=x_0$ 時，$f(x)$ 有最大值 M，則序對 $(\tan x_0,M)=$________.

答案：$(-\frac{5}{12},13)$



解析：$f(x)=\sqrt{5^2+12^2}(\frac{5}{13}\cos x-\frac{12}{13}\sin x)$

$=13(\sin\phi\cos x-\cos\phi\sin x)=13\sin(x-\phi)$,

$\because -1\le\sin(x-\phi)\le 1$,

$\therefore -13\le 13\sin(x-\phi)\le 13$,

$M=13\Rightarrow\sin(x-\phi)=1$

$\Rightarrow x-\phi=2n\pi+\frac{\pi}{2},n\in\mathbb{Z},\Rightarrow x=\phi+2n\pi+\frac{\pi}{2}$

$$\Rightarrow\tan x=\tan(\phi+2n\pi+\frac{\pi}{2})=-\cot\phi=-\frac{5}{12}=\tan x_0,$$

$\therefore (\tan x_0,M)=(-\frac{5}{12},13)$.

10、設 $270° < A < 360°$ 且 $\sqrt{3}\sin A + \cos A = 2\sin 2012°$，若 $A = m°$，則 $m =$ ________。

答案： $298°$

解析： $\sqrt{3}\sin A + \cos A = 2\sin 2012°$

$\Rightarrow\ 2(\frac{\sqrt{3}}{2}\cdot\sin A + \frac{1}{2}\cdot\cos A) = 2\sin 2012°$

$\Rightarrow\ 2(\cos 30°\cdot\sin A + \sin 30°\cdot\cos A) = 2\sin 2012°$

$\Rightarrow\ 2\sin(A+30°) = 2\sin 2012°$

$\Rightarrow\ \sin(A+30°) = \sin 2012° = \sin(360°\times 5 + 212°) = \sin 212° = -\sin 32°$

$\Rightarrow\ \sin(A+30°) = -\sin 32°$,

$\because\ 270° < A < 360°$,

$\therefore \sin(A+30°) = -\sin 32° = \sin(360° - 32°) = \sin 328° \Rightarrow\ A + 30° = 328° \ \Rightarrow\ A = 298°$.

11、試將下列極坐標化為直角坐標：

(1) $A(3,\pi) \Rightarrow$ ________；　(2) $B(4,\frac{2\pi}{3}) \Rightarrow$ ________；

(3) $C(4,-\frac{7\pi}{6}) \Rightarrow$ ________；(4) $D(3,2) \Rightarrow$ ________.

答案：(1) $(-3,0)$ (2) $(-2,2\sqrt{3})$ (3) $(-2\sqrt{3},2)$ (4) $(3\cos 2, 3\sin 2)$

解析：(1) $A(3\cos\pi, 3\sin\pi) = (-3,0)$.

(2) $B(4\cos\frac{2\pi}{3}, 4\sin\frac{2\pi}{3}) = (-2,2\sqrt{3})$.

(3) $C(4\cos(-\frac{7\pi}{6}), 4\sin(-\frac{7\pi}{6})) = (-2\sqrt{3},2)$.

(4) $D(3\cos 2, 3\sin 2)$.

12、　試將下列直角坐標化為極坐標：(取 $r > 0, 0 \le \theta < 2\pi$)

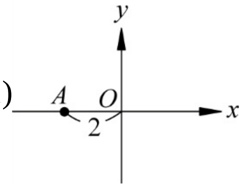
(1) $A(-2,0) \Rightarrow$ ________；　(2) $B(-\sqrt{3},-3) \Rightarrow$ ________；

(3) $C(\sqrt{2},-\sqrt{6}) \Rightarrow$ ________；　(4) $D(\sqrt{3},\sqrt{3}) \Rightarrow$ ________；

(5) $E(4\cos\frac{\pi}{5}, 4\sin\frac{\pi}{5}) \Rightarrow$ ________；(6) $F(1-\sqrt{3}, 1+\sqrt{3}) \Rightarrow$ ________.

答案：(1) $(2,\pi)$　(2) $(2\sqrt{3},\frac{4\pi}{3})$　(3) $(2\sqrt{2},\frac{5\pi}{3})$　(4) $(\sqrt{6},\frac{\pi}{4})$　(5) $(4,\frac{\pi}{5})$　(6) $(2\sqrt{2},\frac{7\pi}{12})$

解析：

(1)


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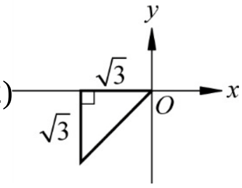
$\sqrt{(-2)^2 + 0^2} = 2,\quad A(2,\pi)$

(2)


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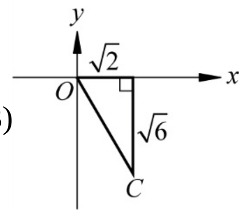
$\sqrt{(-\sqrt{3})^2 + (-3)^2} = 2\sqrt{3},\quad B(2\sqrt{3},\frac{4\pi}{3})$

(3) 

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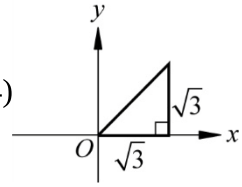
$\sqrt{(\sqrt{2})^2+(-\sqrt{6})^2}=2\sqrt{2},\quad C(2\sqrt{2},\dfrac{5\pi}{3})$

(4) 

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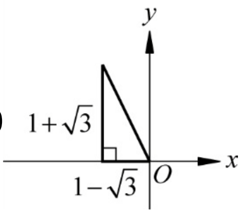
$\sqrt{(\sqrt{3})^2+(\sqrt{3})^2}=2\sqrt{3},\quad D(\sqrt{6},\dfrac{\pi}{4})$

(5) $E(4,\dfrac{\pi}{5})$.

(6) 

$\sqrt{(1+\sqrt{3})^2+(1-\sqrt{3})^2}=2\sqrt{2},\ F(2\sqrt{2},\dfrac{7\pi}{12})$.

13、 $|z|=2|z+1|,\ \mathrm{Arg}(\dfrac{z+1}{z})=\dfrac{\pi}{3}$ ，則 $|z|=$ ________; $\mathrm{Arg}(z)=$ ________.

答案： $\dfrac{2\sqrt{3}}{3};\dfrac{7\pi}{6}$

解析： $\because |z|=2|z+1| \Rightarrow \dfrac{|z+1|}{|z|}=\dfrac{1}{2}=\left|\dfrac{z+1}{z}\right|$

$$\Rightarrow \frac{z+1}{z}=\frac{1}{2}(\cos\frac{\pi}{3}+i\sin\frac{\pi}{3})=\frac{1}{2}(\frac{1}{2}+\frac{\sqrt{3}}{2}i)$$

$$\Rightarrow 1+\frac{1}{z}=\frac{1}{4}+\frac{\sqrt{3}}{4}i$$

$$\Rightarrow \frac{1}{z}=-\frac{3}{4}+\frac{\sqrt{3}}{4}i=\frac{-3+\sqrt{3}i}{4}$$

$$\Rightarrow z=\frac{4(-3-\sqrt{3}i)}{(-3+\sqrt{3}i)(-3-\sqrt{3}i)}=\frac{4(-3-\sqrt{3}i)}{9-3i^2}=\frac{4(-3-\sqrt{3}i)}{9+3}$$

$$=\frac{-3-\sqrt{3}i}{3}=-1-\frac{\sqrt{3}}{3}i=\frac{2\sqrt{3}}{3}(\cos\frac{7\pi}{6}+i\sin\frac{7\pi}{6}),$$

$\therefore |z|=\dfrac{2\sqrt{3}}{3},\ \mathrm{Arg}(z)=\dfrac{7\pi}{6}$.

14、 $\dfrac{(\cos 137^\circ+i\sin 763^\circ)(\cos 317^\circ+i\sin 223^\circ)}{\sin 124^\circ+i\cos(-56^\circ)}=$ ________.

答案： $\dfrac{1}{2}+\dfrac{\sqrt{3}}{2}i$

解析： $\cos 137^\circ+i\sin 763^\circ=-\cos 43^\circ+i\sin 43^\circ=\cos 137^\circ+i\sin 137^\circ$,

$\cos 317^\circ+i\sin 223^\circ=\cos 43^\circ-i\sin 43^\circ=\cos(-43^\circ)+i\sin(-43^\circ)$,

$\sin 124^\circ+i\cos(-56^\circ)=\cos 34^\circ+i\sin 34^\circ$,

$\therefore$ 所求 $=\dfrac{(\cos137^\circ+i\sin137^\circ)[\cos(-43^\circ)+i\sin(-43^\circ)]}{\cos34^\circ+i\sin34^\circ}$

$=\cos[137^\circ+(-43^\circ)-34^\circ]+i\sin[137^\circ+(-43^\circ)-34^\circ]=\cos60^\circ+i\sin60^\circ=\dfrac{1}{2}+\dfrac{\sqrt{3}}{2}i$.

15、 $(\dfrac{\sqrt{3}+i}{\sqrt{2}})^{20}=$ _________.

答案： $-512-512\sqrt{3}i$

解析： $(\dfrac{\sqrt{3}+i}{\sqrt{2}})^{20}=\dfrac{[2(\cos\frac{\pi}{6}+i\sin\frac{\pi}{6})]^{20}}{\sqrt{2}^{20}}=\dfrac{2^{20}[\cos\frac{20\pi}{6}+i\sin\frac{20\pi}{6}]}{2^{10}}$

$=2^{10}[\cos\dfrac{10\pi}{3}+i\sin\dfrac{10\pi}{3}]=1024(\cos\dfrac{4}{3}\pi+i\sin\dfrac{4\pi}{3})$

$=1024(-\dfrac{1}{2}-\dfrac{\sqrt{3}}{2}i)=-512-512\sqrt{3}i$.

16、設 $x=\dfrac{1+\sqrt{3}i}{2}$ ，則 $1+x+x^2+\cdots+x^{13}=$ _________.

答案： $\dfrac{3}{2}+\dfrac{\sqrt{3}}{2}i$

解析： $x=\dfrac{1}{2}+\dfrac{\sqrt{3}}{2}i=\cos\dfrac{\pi}{3}+i\sin\dfrac{\pi}{3}\Rightarrow x^3=1$,

$\therefore 1+x+x^2+\cdots+x^{13}=\dfrac{1(x^{14}-1)}{x-1}$

又 $x^{14}=(x^3)^4\cdot x^2=x^2$ ，　所求 $=\dfrac{x^2-1}{x-1}=x+1=\dfrac{3}{2}+\dfrac{\sqrt{3}}{2}i$.

17、 $(\sin15^\circ+i\cos15^\circ)^{10}=$ _________.

答案： $\dfrac{\sqrt{3}}{2}+\dfrac{1}{2}i$

解析：原式 $=(\cos75^\circ+i\sin75^\circ)^{10}=\cos750^\circ+i\sin750^\circ=\cos30^\circ+i\sin30^\circ=\dfrac{\sqrt{3}}{2}+\dfrac{1}{2}i$.

18、 $\dfrac{\cos(-26^\circ)-i\sin334^\circ}{(\cos137^\circ+i\sin763^\circ)(\cos369^\circ+i\sin171^\circ)}=$ _________.

答案： $-\dfrac{1}{2}-\dfrac{\sqrt{3}}{2}i$

解析：原式 $=\dfrac{\cos26^\circ+i\sin26^\circ}{(\cos137^\circ+i\sin137^\circ)(\cos9^\circ+i\sin9^\circ)}$

$=\cos(26^\circ-137^\circ-9^\circ)+i\sin(26^\circ-137^\circ-9^\circ)=\cos(-120^\circ)+i\sin(-120^\circ)=-\dfrac{1}{2}-\dfrac{\sqrt{3}}{2}i$.

19、$(1+\cos\frac{\pi}{3}+i\sin\frac{\pi}{3})^6=$________.

答案：-27

解析：原式$=(1+\frac{1}{2}+\frac{\sqrt{3}}{2}i)^6=(\frac{3}{2}+\frac{\sqrt{3}}{2}i)^6=[\sqrt{3}(\frac{\sqrt{3}}{2}+\frac{1}{2}i)]^6=3^3(\cos\frac{\pi}{6}+i\sin\frac{\pi}{6})^6$

$=27(\cos\pi+i\sin\pi)=-27$.

20、$f(x)=x^{120}+x^{60}+x^6$, $f(\frac{\sqrt{6}-\sqrt{2}}{4}+\frac{\sqrt{6}+\sqrt{2}}{4}i)=$________.

答案：i

解析：$\frac{\sqrt{6}-\sqrt{2}}{4}+\frac{\sqrt{6}+\sqrt{2}}{4}i=\cos\frac{5\pi}{12}+i\sin\frac{5\pi}{12}$,

$f(\frac{\sqrt{6}-\sqrt{2}}{4}+\frac{\sqrt{6}+\sqrt{2}}{4}i)=(\cos\frac{5\pi}{12}+i\sin\frac{5\pi}{12})^{120}+(\cos\frac{5\pi}{12}+i\sin\frac{5\pi}{12})^{60}+(\cos\frac{5\pi}{12}+i\sin\frac{5\pi}{12})^6$

$=\cos 50\pi+i\sin 50\pi+\cos 25\pi+i\sin 25\pi+\cos\frac{5\pi}{2}+i\sin\frac{5\pi}{2}=1+(-1)+i=i$.

21、$z=\cos\frac{2\pi}{5}+i\sin\frac{2\pi}{5}$, $z^{65}+z^{66}+z^{67}+\cdots+z^{365}=$________.

答案：1

解析：$\because z=\cos\frac{2\pi}{5}+i\sin\frac{2\pi}{5}\Rightarrow z^5=1$.

所求$=(z^5)^{13}+(z^5)^{13}\cdot z+\cdots+(z^5)^{73}$

$=(1+z+z^2+z^3+z^4)+\cdots+(1+z+z^2+z^3+z^4)+1=0+\cdots+0+1=1$.

22、若$z+\frac{1}{z}=\sqrt{3}$，則$z^{12}+\frac{1}{z^{12}}=$________.

答案：2

解析：$z^2-\sqrt{3}z+1=0\Rightarrow z=\frac{\sqrt{3}\pm\sqrt{3-4}}{2}=\frac{\sqrt{3}}{2}\pm\frac{1}{2}i$,

(1) $z=\frac{\sqrt{3}}{2}+\frac{1}{2}i=\cos\frac{\pi}{6}+i\sin\frac{\pi}{6}$,

$\therefore z^{12}+\frac{1}{z^{12}}=z^{12}+z^{-12}=(\cos\frac{\pi}{6}+i\sin\frac{\pi}{6})^{12}+(\cos\frac{\pi}{6}+i\sin\frac{\pi}{6})^{-12}$

$=\cos 2\pi+i\sin 2\pi+\cos(-2\pi)+i\sin(-2\pi)=1+0+1+0=2$.

(2) $z=\frac{\sqrt{3}}{2}-\frac{1}{2}i=\cos(-\frac{\pi}{6})+i\sin(-\frac{\pi}{6})$,

$z^{12}+\frac{1}{z^{12}}=z^{12}+z^{-12}=[\cos(-\frac{\pi}{6})+i\sin(-\frac{\pi}{6})]^{12}+[\cos(-\frac{\pi}{6})+i\sin(-\frac{\pi}{6})]^{-12}$

$=\cos(-2\pi)+i\sin(-2\pi)+\cos 2\pi+i\sin 2\pi=1+1=2$.

$\therefore z^{12}+\frac{1}{z^{12}}=2$.

23、若 $z=\dfrac{(1-\sqrt{2}i)^4(1+i)^8}{(\sqrt{3}+i)}$ ，則 $|z|=$________.

答案：72

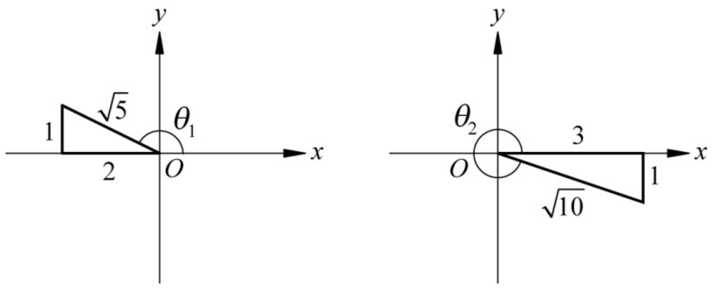
解析：$|z|=\dfrac{|1-\sqrt{2}i|^4|1+i|^8}{|\sqrt{3}+i|}=\dfrac{\sqrt{3}^4\cdot\sqrt{2}^8}{2}=\dfrac{9\cdot 16}{2}=72$.

24、設 $z_1=-2+i$, $z_2=3-i$, $\mathrm{Arg}(z_1)=\theta_1$, $\mathrm{Arg}(z_2)=\theta_2$, $0<\theta_1,\theta_2<2\pi$ ，則

(1) $\cos(\theta_1+\theta_2)=$________；(2) $\theta_1+\theta_2=$________.

答案：(1) $-\dfrac{\sqrt{2}}{2}$ (2) $\dfrac{3\pi}{4}$

解析：



$\cos\theta_1=-\dfrac{2}{\sqrt{5}},\sin\theta_1=\dfrac{1}{\sqrt{5}}$, $\cos\theta_2=\dfrac{3}{\sqrt{10}},\sin\theta_2=-\dfrac{1}{\sqrt{10}}$,

(1) $\cos(\theta_1+\theta_2)=\cos\theta_1\cos\theta_2-\sin\theta_1\sin\theta_2=-\dfrac{2}{\sqrt{5}}\dfrac{3}{\sqrt{10}}+\dfrac{1}{\sqrt{5}}\dfrac{1}{\sqrt{10}}=\dfrac{-6}{5\sqrt{2}}+\dfrac{1}{5\sqrt{2}}=\dfrac{-1}{\sqrt{2}}=-\dfrac{\sqrt{2}}{2}$.

(2) $\sin(\theta_1+\theta_2)=\sin\theta_1\cos\theta_2+\sin\theta_2\cos\theta_1=\dfrac{1}{\sqrt{5}}\cdot\dfrac{3}{\sqrt{10}}+\dfrac{-1}{\sqrt{10}}\cdot\dfrac{-2}{\sqrt{5}}=\dfrac{3}{5\sqrt{2}}+\dfrac{2}{5\sqrt{2}}=\dfrac{1}{\sqrt{2}}=\dfrac{\sqrt{2}}{2}$,

$\because\sin(\theta_1+\theta_2)>0$, $\cos(\theta_1+\theta_2)<0$, $\therefore\theta_1+\theta_2=\dfrac{3\pi}{4}$.

25、以 $x^8+x^4+1=0$ 之各根爲頂點之正多邊形面積爲________.

答案：$\sqrt{3}+1$

解析：$\because x^8+x^4+1=0\Rightarrow(x^4-1)(x^8+x^4+1)=0\Rightarrow x^{12}-1=0$

$\Rightarrow x^{12}=1=\cos 0+i\sin 0=\cos(2k\pi)+i\sin(2k\pi), k\in\mathbb{Z}$

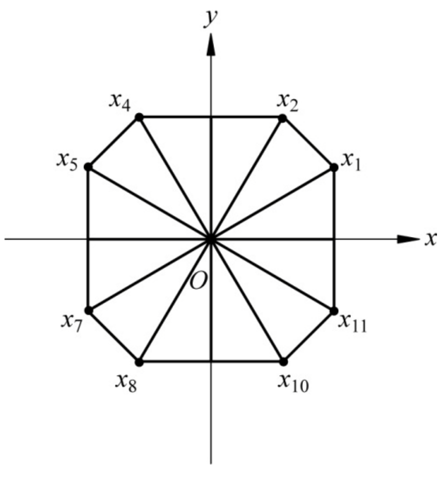
$\Rightarrow x_k=[\cos(2k\pi)+i\sin(2k\pi)]^{\frac{1}{12}}, k=0,1,2,\cdots,11$

$=\cos\dfrac{2k\pi}{12}+i\sin\dfrac{2k\pi}{12}$,

又 $x\neq\pm1,\pm i$,

$\therefore x_k=\cos\dfrac{k\pi}{6}+i\sin\dfrac{k\pi}{6}, k=1,2,4,5,7,8,10,11$,

面積 $=4\cdot(\dfrac{1}{2}\cdot1\cdot1\cdot\sin 60^\circ)+4\cdot(\dfrac{1}{2}\cdot1\cdot1\cdot\sin 30^\circ)=\sqrt{3}+1$.



26、設複數 $z=1-i$；若 $1+z+z^2+\cdots+z^9=a+bi$，其中 a, b 爲實數，則 $a=$______，$b=$______

答案：32；-1

解析：$1+z+z^2+\cdots\cdots+z^9=\dfrac{1\cdot(1-z^{10})}{1-z}=\dfrac{1-(1-i)^{10}}{1-(1-i)}=\dfrac{1-[(1-i)^2]^5}{i}=\dfrac{1-[-2i]^5}{i}$

$$=\frac{1-\{-32i^5\}}{i}=\frac{1+32i}{i}=32-i$$

$\therefore a=32$，$b=-1$