高雄市明誠中學 高三數學平時測驗					日期：100.11.17
範圍	第 6 回三角函數(2)	班級	三年 班	姓名	
		座號			

一、填充題(每題 10 分)

1、 試求：(1) $\sin\frac{\pi}{8}=$________；(2) $\cos\frac{\pi}{16}=$________.

答案：(1) $\frac{\sqrt{2-\sqrt{2}}}{2}$ (2) $\frac{\sqrt{2+\sqrt{2+\sqrt{2}}}}{2}$

解析：(1) $\sin\frac{\pi}{8}=\sin\frac{\frac{\pi}{4}}{2}=\sqrt{\frac{1-\cos\frac{\pi}{4}}{2}}=\sqrt{\frac{1-\frac{\sqrt{2}}{2}}{2}}=\sqrt{\frac{\frac{2-\sqrt{2}}{2}}{2}}=\frac{\sqrt{2-\sqrt{2}}}{2}$.

(2) $\cos\frac{\pi}{16}=\cos\frac{\frac{\pi}{8}}{2}=\sqrt{\frac{1+\cos\frac{\pi}{8}}{2}}=\sqrt{\frac{1+\frac{\sqrt{2+\sqrt{2}}}{2}}{2}}=\sqrt{\frac{2+\sqrt{2+\sqrt{2}}}{4}}=\frac{\sqrt{2+\sqrt{2+\sqrt{2}}}}{2}$.

2、 設 $x^2+px+q=0$ 之兩根爲 $\sin\theta,\cos\theta$，試求 $2\sin^2\frac{\theta}{2}(\cos\frac{\theta}{2}-\sin\frac{\theta}{2})^2=$________.

答案：$1+p+q$

解析：$\because \sin\theta+\cos\theta=-p$, $\sin\theta\cos\theta=q$.

$2\sin^2\frac{\theta}{2}(\cos\frac{\theta}{2}-\sin\frac{\theta}{2})^2$

$=2\cdot\frac{1-\cos\theta}{2}\cdot(\cos^2\frac{\theta}{2}-2\sin\frac{\theta}{2}\cos\frac{\theta}{2}+\sin^2\frac{\theta}{2})$

$=(1-\cos\theta)(1-\sin\theta)=1-(\sin\theta+\cos\theta)+\sin\theta\cos\theta=1-(-p)+q=1+p+q$.

3、 若 $\cos 4\theta=\frac{1}{2}$，求 $\sin^6\theta+\cos^6\theta=$________.

答案：$\frac{13}{16}$

解析：$\cos 4\theta=1-2\sin^2 2\theta=\frac{1}{2}\Rightarrow 2\sin^2 2\theta=\frac{1}{2}\Rightarrow \sin^2 2\theta=\frac{1}{4}\Rightarrow \sin 2\theta=\pm\frac{1}{2}$,

$\sin^6\theta+\cos^6\theta=(\sin^2\theta+\cos^2\theta)^3-3\sin^2\theta\cos^2\theta(\sin^2\theta+\cos^2\theta)$

$=1-\frac{3}{4}(4\sin^2\theta\cos^2\theta)=1-\frac{3}{4}\sin^2 2\theta=1-\frac{3}{4}\cdot\frac{1}{4}=\frac{13}{16}$.

4、 若 $\pi<\theta<\frac{3\pi}{2},\sin\theta=-\frac{3}{5}$，求 $\tan\frac{\theta}{2}=$________.

答案：-3

解析：$\cos\theta=-\frac{4}{5}\Rightarrow \tan\frac{\theta}{2}=\frac{1-\cos\theta}{\sin\theta}=\frac{1-(-\frac{4}{5})}{-\frac{3}{5}}=\frac{\frac{9}{5}}{-\frac{3}{5}}=-3$.

5、 若 $\tan\frac{\theta}{2}=3$，求 $\cos2\theta+\sin2\theta=$________.

答案：$\frac{-17}{25}$

解析：$\tan\theta=\frac{2\tan\frac{\theta}{2}}{1-\tan^2\frac{\theta}{2}}=\frac{2\times3}{1-9}=\frac{6}{-8}=-\frac{3}{4}$,

$$\cos2\theta+\sin2\theta=\frac{1-\tan^2\theta}{1+\tan^2\theta}+\frac{2\tan\theta}{1+\tan^2\theta}=\frac{1-(-\frac{3}{4})^2+2\cdot(-\frac{3}{4})}{1+(-\frac{3}{4})^2}=\frac{1-\frac{9}{16}-\frac{3}{2}}{1+\frac{9}{16}}=\frac{\frac{16-9-24}{16}}{\frac{25}{16}}=\frac{-17}{25}.$$

6、 試求：(1) $\sin18°=$________；(2) $\cos36°=$________.

答案：(1) $\frac{\sqrt{5}-1}{4}$ (2) $\frac{\sqrt{5}+1}{4}$

解析：(1) $\sin18°$

令 $\theta=18°\Rightarrow5\theta=90°\Rightarrow3\theta=90°-2\theta$

$\Rightarrow\sin3\theta=\sin(90°-2\theta)$

$\Rightarrow3\sin\theta-4\sin^3\theta=\cos2\theta=1-2\sin^2\theta$

$\Rightarrow4\sin^3\theta-2\sin^2\theta-3\sin\theta+1=0$ (令 $t=\sin\theta$)

$\Rightarrow4t^3-2t^2-3t+1=0\Rightarrow(t-1)(4t^2+2t-1)=0\Rightarrow t=1,\frac{-2\pm\sqrt{4+16}}{2\cdot4}$

$\Rightarrow\sin\theta=\sin18°=1,\frac{-1\pm\sqrt{5}}{4}$, $\therefore\sin18°=\frac{\sqrt{5}-1}{4}$.

(2) $\cos36°$

$\Rightarrow\theta=36°\Rightarrow5\theta=180°\Rightarrow3\theta=180°-2\theta$

$\Rightarrow\cos3\theta=\cos(180°-2\theta)=-\cos2\theta$

$\Rightarrow4\cos^3\theta-3\cos\theta=-(2\cos^2\theta-1)$

$\Rightarrow4\cos^3\theta+2\cos^2\theta-3\cos\theta-1=0$

$\Rightarrow4y^3+2y^2-3y-1=0,\Rightarrow(y+1)(4y^2-2y-1)=0\Rightarrow y=-1,\frac{2\pm\sqrt{4+16}}{8}$

$\Rightarrow\cos\theta=\cos36°=-1,\frac{1\pm\sqrt{5}}{4}>0$, $\therefore\cos36°=\frac{\sqrt{5}+1}{4}$.

7、 試求：$\cos\frac{2\pi}{7}\cdot\cos\frac{4\pi}{7}\cdot\cos\frac{6\pi}{7}=$________.

答案：$\frac{1}{8}$

解析：$\because\cos\frac{6\pi}{7}=\cos(2\pi-\frac{8\pi}{7})=\cos\frac{8\pi}{7}$,

設 $p=\cos\frac{2\pi}{7}\cos\frac{4\pi}{7}\cos\frac{8\pi}{7}$，

$$2^3\sin\frac{2\pi}{7}p=2^2(2\sin\frac{2\pi}{7}\cos\frac{2\pi}{7})\cos\frac{4\pi}{7}\cos\frac{8\pi}{7}$$

$$=2(2\sin\frac{4\pi}{7}\cdot\cos\frac{4\pi}{7})\cos\frac{8\pi}{7}$$

$$=2\sin\frac{8\pi}{7}\cos\frac{8\pi}{7}$$

$$=\sin\frac{16\pi}{7}$$

$$=\sin(2\pi+\frac{2\pi}{7}),$$

$$=\sin\frac{2\pi}{7}$$

$$\therefore p=\frac{1}{8}.$$

8、 $\tan 9°-\tan 27°+\tan 81°-\tan 63°=$________.

答案：4

解析：原式 $=\tan 9°+\tan 81°-(\tan 27°+\tan 63°)$

$$=(\tan 9°+\cot 9°)-(\tan 27°+\cot 27°)=\frac{1}{\cos 9°\sin 9°}-\frac{1}{\cos 27°\sin 27°}\quad(\text{公式})$$

$$=\frac{2}{2\sin 9°\cos 9°}-\frac{2}{2\sin 27°\cos 27°}=\frac{2}{\sin 18°}-\frac{2}{\sin 54°}=\frac{2}{\sin 18°}-\frac{2}{\cos 36°}$$

$$=\frac{2}{\frac{\sqrt{5}-1}{4}}-\frac{2}{\frac{\sqrt{5}+1}{4}}\quad(\text{參閱 NO.6})$$

$$=\frac{8}{\sqrt{5}-1}-\frac{8}{\sqrt{5}+1}=2(\sqrt{5}+1)-2(\sqrt{5}-1)=4.$$

$$\left(\because \tan 9°+\cot 9°=\frac{\sin 9°}{\cos 9°}+\frac{\cos 9°}{\sin 9°}=\frac{\sin^2 9°+\cos^2 9°}{\sin 9°\cos 9°}=\frac{1}{\sin 9°\cos 9°}\right.$$

$$\left.\text{同理 }\tan 27°+\cot 27°=\frac{1}{\sin 27°\cos 27°}\right)$$

9、 若 θ 在第三象限，且 $\cos\theta=-\frac{4}{5}$，則 $\cos(\theta+\frac{\pi}{4})=$________.

答案：$\frac{-\sqrt{2}}{10}$

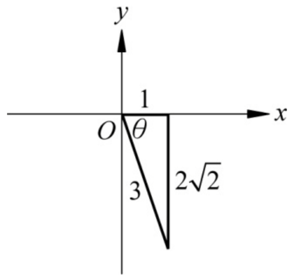
解析：θ 在第三象限，且 $\cos\theta=-\frac{4}{5}\Rightarrow\sin\theta=-\frac{3}{5}$,

$$\cos(\theta+\frac{\pi}{4})=\cos\theta\cos\frac{\pi}{4}-\sin\theta\sin\frac{\pi}{4}=(-\frac{4}{5})(\frac{\sqrt{2}}{2})-(-\frac{3}{5})(\frac{\sqrt{2}}{2})=\frac{-4\sqrt{2}}{10}+\frac{3\sqrt{2}}{10}=\frac{-\sqrt{2}}{10}.$$

10、若 $180^\circ<\theta<360^\circ$，且 $\cos\theta=\frac{1}{3}$，則 $\sin\theta=$________, $\tan\theta=$________.

答案：$-\frac{2\sqrt{2}}{3};-2\sqrt{2}$

解析：$180^\circ < \theta < 360^\circ$，且 $\cos\theta = \dfrac{1}{3} \Rightarrow \theta$ 在第四象限



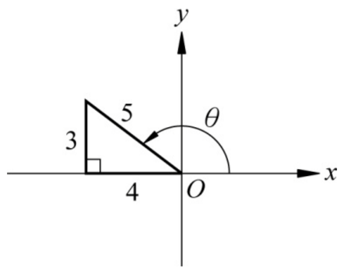
$\sin\theta = -\dfrac{2\sqrt{2}}{3}$, $\tan\theta = -\dfrac{2\sqrt{2}}{1} = -2\sqrt{2}$.

11、 (1)若 $0^\circ < \theta < 180^\circ$，且 $\cos\theta = -\dfrac{4}{5}$，則 $\sin\theta =$ ________, $\tan\theta =$ ________.

(2)若 $90^\circ < \theta < 270^\circ$，且 $\tan\theta = 3$，則 $\sin\theta =$ ________, $\cos\theta =$ ________.

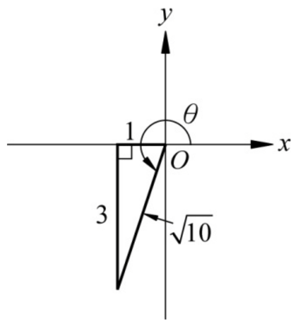
答案：(1) $\dfrac{3}{5}; -\dfrac{3}{4}$ (2) $-\dfrac{3\sqrt{10}}{10}; -\dfrac{\sqrt{10}}{10}$

解析：



(1) $0^\circ < \theta < 180^\circ$，且 $\cos\theta = -\dfrac{4}{5} \Rightarrow \sin\theta = \dfrac{3}{5}$, $\tan\theta = -\dfrac{3}{4}$.

(2) $90^\circ < \theta < 270^\circ$，且 $\tan\theta = 3 \Rightarrow \sin\theta = -\dfrac{3}{\sqrt{10}} = -\dfrac{3\sqrt{10}}{10}$,



$\cos\theta = -\dfrac{1}{\sqrt{10}} = -\dfrac{\sqrt{10}}{10}$.

12、 求值：(1) $\sin 2025^\circ =$ ________. (2) $\cot(-1035^\circ) =$ ________.

答案：(1) $-\dfrac{\sqrt{2}}{2}$ (2) 1

解析：(1) $\sin 2025^\circ = \sin(90^\circ \times 22 + 45^\circ) = -\sin 45^\circ = -\dfrac{\sqrt{2}}{2}$.

(2) $\cot(-1035^\circ) = -\cot 1035^\circ = -\cot(90^\circ \times 11 + 45^\circ) = +\tan 45^\circ = 1$.

13、若 $\sin(630^\circ - \theta^\circ) = \dfrac{1}{2}$，且 θ 在第三象限，則 $\cos(\theta - 1260^\circ) =$ ________.

答案：$\dfrac{1}{2}$

解析：$\sin(630^\circ - \theta) = \sin(90^\circ \times 7 - \theta) = -\cos\theta = \dfrac{1}{2} \Rightarrow \cos\theta = -\dfrac{1}{2}$，且 θ 在第三象限，

$\therefore$ $\cos(\theta - 1260^\circ) = \cos(1260^\circ - \theta) = \cos(90^\circ \times 14 - \theta) = -\cos\theta = \dfrac{1}{2}$.

14、化簡：$\tan(180^\circ - \theta) \cdot \cot(-180^\circ - \theta) - \sin^2(180^\circ + \theta) =$ ________.

答案：$\cos^2\theta$

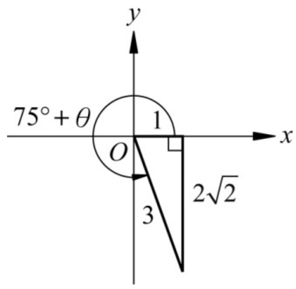
解析：原式 $= (-\tan\theta) \cdot (-\cot\theta) - [-\sin\theta]^2 = 1 - \sin^2\theta = \cos^2\theta$.

15、若 $\cos(75^\circ + \theta) = \dfrac{1}{3}$，其中 θ 爲第三象限角，則 $\cos(105^\circ - \theta) + \sin(\theta - 105^\circ) =$ ________.

答案：$\dfrac{2\sqrt{2} - 1}{3}$

解析：$\because 360^\circ\cdot n+180^\circ<\theta<360^\circ\cdot n+270^\circ, n\in\mathbb{Z}$

$\Rightarrow 360^\circ\cdot n+225^\circ<\theta+75^\circ<360^\circ\cdot n+345^\circ, n\in\mathbb{Z},$



又 $\cos(75^\circ+\theta)=\dfrac{1}{3}\Rightarrow 75^\circ+\theta$ 爲第四象限角

所求 $\cos(105^\circ-\theta)+\sin(\theta-105^\circ)$

$=\cos[180^\circ-(75^\circ+\theta)]+\sin[(75^\circ+\theta)-180^\circ]$

$=-\cos(75^\circ+\theta)-\sin(75^\circ+\theta)=-\dfrac{1}{3}-(-\dfrac{2\sqrt{2}}{3})=\dfrac{2\sqrt{2}-1}{3}.$

16、若 $\sin\theta+\cos\theta=\dfrac{\sqrt{3}-1}{2}$ ，且 $0<\theta<180^\circ$ ，則 $\tan\theta=$________.

答案：$-\sqrt{3}$

解析：$\because \begin{cases}\sin\theta+\cos\theta=\dfrac{\sqrt{3}-1}{2}\\ \sin^2\theta+\cos^2\theta=1\end{cases},$

$(\sin\theta+\cos\theta)^2=\sin^2\theta+2\sin\theta\cos\theta+\cos^2\theta$

$\Rightarrow (\dfrac{\sqrt{3}-1}{2})^2=1+2\sin\theta\cos\theta$

$\Rightarrow \dfrac{4-2\sqrt{3}}{4}=1+2\sin\theta\cos\theta\Rightarrow 2\sin\theta\cos\theta=-\dfrac{\sqrt{3}}{2},$

$\Rightarrow (\sin\theta-\cos\theta)^2=\sin^2\theta+\cos^2\theta-2\sin\theta\cos\theta=1+\dfrac{\sqrt{3}}{2}=\dfrac{2+\sqrt{3}}{2}$

$\Rightarrow \sin\theta-\cos\theta=\sqrt{\dfrac{4+2\sqrt{3}}{4}}=\dfrac{\sqrt{3}+1}{2},$

$\therefore \begin{cases}\sin\theta+\cos\theta=\dfrac{\sqrt{3}-1}{2}\\ \sin\theta-\cos\theta=\dfrac{\sqrt{3}+1}{2}\end{cases},$ 得 $\sin\theta=\dfrac{\sqrt{3}}{2},\cos\theta=-\dfrac{1}{2}$ 故 $\tan\theta=\dfrac{\frac{\sqrt{3}}{2}}{\frac{-1}{2}}=-\sqrt{3}.$

17、若 $2\sin\theta+\cos\theta=0$,則 $2\sin^2\theta-3\sin\theta\cos\theta-5\cos^2\theta=$________.

答案：$-\dfrac{12}{5}$

解析：$2\sin\theta=-\cos\theta\Rightarrow \dfrac{\sin\theta}{\cos\theta}=-\dfrac{1}{2}=\tan\theta,$

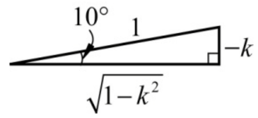
所求 $=\cos^2\theta(2\dfrac{\sin^2\theta}{\cos^2\theta}-3\dfrac{\sin\theta}{\cos\theta}-5)=\dfrac{1}{\sec^2\theta}(2\tan^2\theta-3\tan\theta-5)$

$=\dfrac{2\tan^2\theta-3\tan\theta-5}{\tan^2\theta+1}=\dfrac{2\cdot\frac{1}{4}-3(-\frac{1}{2})-5}{\frac{1}{4}+1}=\dfrac{\frac{1}{2}+\frac{3}{2}-5}{\frac{5}{4}}=\dfrac{-3}{\frac{5}{4}}=\dfrac{-12}{5}.$

18、設$\cos(-100^\circ)=k$，則$\tan190^\circ=$________.

答案：$\dfrac{-k\sqrt{1-k^2}}{1-k^2}$

解析：$\cos(-100^\circ)=-\cos100^\circ=-\sin10^\circ=k<0\Rightarrow\sin10^\circ=-k=\dfrac{-k}{1}>0$

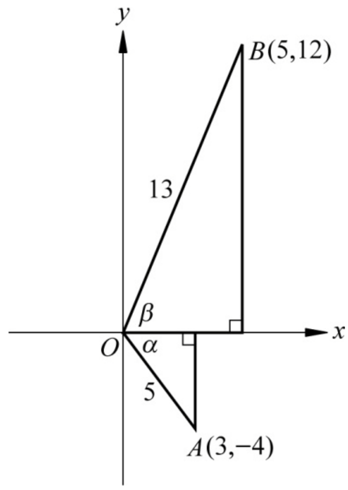


$\tan190^\circ=\tan(180^\circ+10^\circ)=\tan10^\circ=\dfrac{-k}{\sqrt{1-k^2}}=\dfrac{-k\sqrt{1-k^2}}{1-k^2}$.

19、坐標平面上，O表原點，$A(3,-4),B(5,12)$，則

(1)$\cos\angle AOB=$________；(2)$\sin\angle AOB=$________.

答案：(1)$-\dfrac{33}{65}$　(2)$\dfrac{56}{65}$



解析：$\overline{OA}=\sqrt{3^2+(-4)^2}=5;\quad\overline{OB}=\sqrt{5^2+12^2}=13$

$\because\sin\alpha=\dfrac{4}{5},\quad\cos\alpha=\dfrac{3}{5},\qquad\sin\beta=\dfrac{12}{13},\quad\cos\beta=\dfrac{5}{13},$

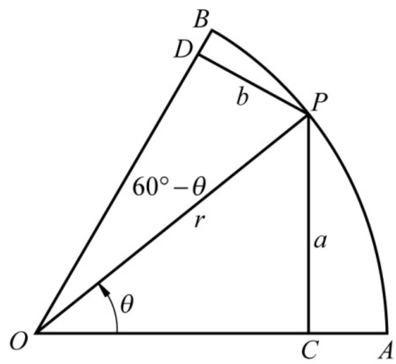
$\therefore\cos\angle AOB=\cos(\alpha+\beta)=\cos\alpha\cos\beta-\sin\alpha\sin\beta$

$=\dfrac{3}{5}\cdot\dfrac{5}{13}-\dfrac{4}{5}\cdot\dfrac{12}{13}=\dfrac{15-48}{65}=-\dfrac{33}{65}$.

$\sin\angle AOB=\sin(\alpha+\beta)=\sin\alpha\cos\beta+\cos\alpha\sin\beta$

$=\dfrac{4}{5}\cdot\dfrac{5}{13}+\dfrac{3}{5}\cdot\dfrac{12}{13}=\dfrac{20+36}{65}=\dfrac{56}{65}$.

20、在扇形OAB中，O為圓心，$\overline{OA}=\overline{OB}=r$為半徑，$\angle AOB=60^\circ$.若$P$為圓弧$\overset{\frown}{AB}$上一點，而$P$至$\overline{OA}$的距離為$a$，$P$至$\overline{OB}$的距離為$b$，則$r=$________.(以$a$, b表示之)



答案：$\dfrac{2\sqrt{3a^2+3ab+3b^2}}{3}$

解析：$\overline{OC}=\sqrt{r^2-a^2},\sin\theta=\dfrac{a}{r},\cos\theta=\dfrac{\sqrt{r^2-a^2}}{r},$

$\sin(60^\circ-\theta)=\dfrac{b}{r}\Rightarrow\dfrac{\sqrt3}{2}\cos\theta-\dfrac{1}{2}\sin\theta=\dfrac{b}{r}$

$\Rightarrow\dfrac{\sqrt3}{2}\cdot\dfrac{\sqrt{r^2-a^2}}{r}-\dfrac{1}{2}\cdot\dfrac{a}{r}=\dfrac{b}{r}$

$\Rightarrow\sqrt3\cdot\sqrt{r^2-a^2}-a=2b$

$\Rightarrow\sqrt3\cdot\sqrt{r^2-a^2}=a+2b$　平方$\Rightarrow3(r^2-a^2)=a^2+4ab+4b^2$

$\Rightarrow3r^2=4(a^2+ab+b^2)\Rightarrow r^2=\dfrac{4(a^2+ab+b^2)}{3}$

$\Rightarrow r=\dfrac{2\sqrt{a^2+ab+b^2}}{\sqrt3}=\dfrac{2\sqrt3\sqrt{a^2+ab+b^2}}{3}=\dfrac{2\sqrt{3a^2+3ab+3b^2}}{3}$.

21、 $\sin 16^\circ \cdot \cos 316^\circ - \sin 224^\circ \cdot \cos 344^\circ =$________.

答案：$\dfrac{\sqrt{3}}{2}$

解析：$316^\circ = 90^\circ \times 4 - 44^\circ,\quad 224^\circ = 90^\circ \times 2 + 44^\circ,\quad 344^\circ = 90^\circ \times 4 - 16^\circ$

原式$= \sin 16^\circ \cos 44^\circ - (-\sin 44^\circ)\cos 16^\circ$

$$= \sin 16^\circ \cos 44^\circ + \cos 16^\circ \sin 44^\circ = \sin(44^\circ + 16^\circ) = \sin 60^\circ = \frac{\sqrt{3}}{2}.$$

22、若$\sin\alpha + \sin\beta = \dfrac{3}{5},\quad \cos\alpha + \cos\beta = \dfrac{4}{5}$，試求$\cos(\alpha - \beta) =$________.

答案：$-\dfrac{1}{2}$

解析：$(\sin\alpha + \sin\beta)^2 + (\cos\alpha + \cos\beta)^2 = \dfrac{9}{25} + \dfrac{16}{25}$

$\Rightarrow (\sin^2\alpha + 2\sin\alpha\sin\beta + \sin^2\beta) + (\cos^2\alpha + 2\cos\alpha\cos\beta + \cos^2\beta) = 1$

$\Rightarrow 2 + 2(\cos\alpha\cos\beta + \sin\alpha\sin\beta) = 1$

$\Rightarrow 2\cos(\alpha - \beta) = -1 \Rightarrow \cos(\alpha - \beta) = -\dfrac{1}{2}.$

23、若$\dfrac{\pi}{4} < \alpha < \dfrac{3\pi}{4}, 0 < \beta < \dfrac{\pi}{4}$，且$\cos(\dfrac{\pi}{4} - \alpha) = \dfrac{3}{5}, \sin(\dfrac{3\pi}{4} + \beta) = \dfrac{5}{13}$，則$\sin(\alpha + \beta) =$________.

答案：$\dfrac{56}{65}$

解析：$\because -\dfrac{3\pi}{4} < -\alpha < -\dfrac{\pi}{4} \Rightarrow -\dfrac{\pi}{2} < \dfrac{\pi}{4} - \alpha < 0 \Rightarrow \sin(\dfrac{\pi}{4} - \alpha) = -\dfrac{4}{5}$

$\because 0 < \beta < \dfrac{\pi}{4},\ \therefore \dfrac{3\pi}{4} < \beta + \dfrac{3\pi}{4} < \pi \Rightarrow \cos(\dfrac{3\pi}{4} + \beta) = -\dfrac{12}{13},$

$\cos[(\dfrac{3\pi}{4} + \beta) - (\dfrac{\pi}{4} - \alpha)] = \cos(\dfrac{3\pi}{4} + \beta)\cos(\dfrac{\pi}{4} - \alpha) + \sin(\dfrac{3\pi}{4} + \beta)\sin(\dfrac{\pi}{4} - \alpha)$

$$= (-\frac{12}{13}) \times \frac{3}{5} + \frac{5}{13} \times (-\frac{4}{5}) = -\frac{56}{65}$$

$\Rightarrow \cos[\dfrac{\pi}{2} + (\alpha + \beta)] = -\dfrac{56}{65} \Rightarrow -\sin(\alpha + \beta) = -\dfrac{56}{65} \quad \Rightarrow \sin(\alpha + \beta) = \dfrac{56}{65}.$

24、若$13\sin\alpha + 5\cos\beta = 9, 13\cos\alpha + 5\sin\beta = 15$，則$\sin(\alpha + \beta) =$________.

答案：$\dfrac{56}{65}$

解析：$\begin{cases} (13\sin\alpha + 5\cos\beta)^2 = 81 \\ (13\cos\alpha + 5\sin\beta)^2 = 225 \end{cases} \Rightarrow \begin{cases} 169\sin^2\alpha + 130\sin\alpha\cos\beta + 25\cos^2\beta = 81 \cdots\cdots ① \\ 169\cos^2\alpha + 130\cos\alpha\sin\beta + 25\sin^2\beta = 225 \cdots\cdots ② \end{cases}$

①+② 得$169(\sin^2\alpha + \cos^2\alpha) + 130\sin(\alpha + \beta) + 25(\cos^2\beta + \sin^2\beta) = 306$

$\Rightarrow 130\sin(\alpha + \beta) + 169 + 25 = 306$

$\Rightarrow 130\sin(\alpha + \beta) = 112 \Rightarrow \sin(\alpha + \beta) = \dfrac{56}{65}.$

25、當$-\frac{\pi}{4}\le x\le\frac{\pi}{3}$時，試求$f(x)=-\sec^2 x+2\tan x+1$的最大值爲________.

答案：1

解析：$f(x)=-(1+\tan^2 x)+2\tan x+1=-\tan^2 x+2\tan x=-(\tan^2 x-2\tan x+1)+1=-(\tan x-1)^2+1$

當$\tan x=1, x=\frac{\pi}{4}$時，$f(x)$有最大值 1.

26、$f(x)=3\cos x-4\sin x+1$，且x爲實數，試求$f(x)$之最大值爲________，最小值爲________.

答案：$6;-4$

解析：$f(x)=5(\frac{3}{5}\cos x-\frac{4}{5}\sin x)+1=5(\sin\phi\cos x-\cos\phi\sin x)+1$，其中$\sin\phi=\frac{3}{5},\quad \cos\phi=\frac{4}{5}$

$=5\sin(\phi-x)+1$,

$\because -1\le\sin(\phi-x)\le 1\Rightarrow -5\le 5\sin(\phi-x)\le 5\Rightarrow -4\le 5\sin(\phi-x)+1\le 6$,

最大值$=6$，最小值$=-4$.

27、$f(x)=2\sin(x-\frac{\pi}{6})+2\cos x-1, -\frac{\pi}{6}\le x\le\frac{5\pi}{6}$，則

(1)當$x=$________時，$f(x)$之最大值爲________；

(2)當$x=$________時，$f(x)$之最小值爲________.

答案：(1)$\frac{\pi}{3}$;1;　(2)$\frac{-\pi}{6},\frac{5\pi}{6}$;$-1$

解析：$f(x)=2\sin(x-\frac{\pi}{6})+2\cos x-1$,

$=2[\sin x\cos\frac{\pi}{6}-\cos x\sin\frac{\pi}{6}]+2\cos x-1$

$=2[\sin x\cdot\frac{\sqrt{3}}{2}-\frac{1}{2}\cos x]+2\cos x-1$

$=\sqrt{3}\sin x+\cos x-1=2[\frac{\sqrt{3}}{2}\sin x+\frac{1}{2}\cos x]-1$

$=2[\cos\frac{\pi}{6}\sin x+\sin\frac{\pi}{6}\cos x]-1=2\sin(x+\frac{\pi}{6})-1$,

$\because\ \frac{-\pi}{6}\le x\le\frac{5\pi}{6}\Rightarrow 0\le x+\frac{\pi}{6}\le\pi\Rightarrow 0\le\sin(x+\frac{\pi}{6})\le 1$

$\Rightarrow 0\le 2\sin(x+\frac{\pi}{6})\le 2\Rightarrow -1\le 2\sin(x+\frac{\pi}{6})-1\le 1$,

(1)當$\sin(x+\frac{\pi}{6})=1$即$x+\frac{\pi}{6}=\frac{\pi}{2}\Rightarrow x=\frac{\pi}{3}$，$f(x)$有最大值$=1$

(2)當$\sin(x+\frac{\pi}{6})=0$即$x+\frac{\pi}{6}=0$或π，$x=-\frac{\pi}{6},\frac{5\pi}{6}$，$f(x)$有最小值$=-1$.

28、若 x 為實數，則 $\dfrac{1+\sin x}{3+\cos x}$ 之最大值為________，最小值為________.

答案：$\dfrac{3}{4};0$

解析：設 $y=\dfrac{1+\sin x}{3+\cos x}\Rightarrow 1+\sin x=3y+y\cos x$

$$\Rightarrow \sin x-y\cos x=3y-1$$

$$\Rightarrow \sqrt{1+y^2}(\sin x\cdot\frac{1}{\sqrt{1+y^2}}-\cos x\cdot\frac{y}{\sqrt{1+y^2}})=3y-1$$

$\Rightarrow \sqrt{1+y^2}\sin(x-\theta)=3y-1$，其中 $\cos\theta=\dfrac{1}{\sqrt{1+y^2}},\quad \sin\theta=\dfrac{y}{\sqrt{1+y^2}}$

$$\Rightarrow \sin(x-\theta)=\frac{3y-1}{\sqrt{1+y^2}}$$

又 $-1\le\sin(x-\theta)\le 1\Rightarrow -1\le\dfrac{3y-1}{\sqrt{1+y^2}}\le 1$

$$\Rightarrow\left|\frac{3y-1}{\sqrt{1+y^2}}\right|\le 1$$

$$\Rightarrow|3y-1|\le\sqrt{1+y^2}$$

$$\Rightarrow (3y-1)^2\le 1+y^2\Rightarrow 4y^2-3y\le 0$$

$\Rightarrow y(4y-3)\le 0\quad\Rightarrow 0\le y\le\dfrac{3}{4}$ ， 最大值 $=\dfrac{3}{4}$，最小值 $=0$.

29、若 θ 在第二象限，試求：$\tan\theta+\cot\theta$ 之最大值為________.

答案：-2

解析：$\because\theta$ 在第二象限 $\Rightarrow\tan\theta<0,\cot\theta<0$

$\dfrac{(-\tan\theta)+(-\cot\theta)}{2}\ge\sqrt{(-\tan\theta)(-\cot\theta)}$ (算幾不等式)

$\Rightarrow -(\tan\theta+\cot\theta)\ge 2\cdot 1\Rightarrow\tan\theta+\cot\theta\le -2$， $\therefore$最大值為 -2.

30、 $f(x)=\sin^2 x+2\sin x\cos x+3\cos^2 x$，則 $f(x)$ 之最大值為________，最小值為________.

答案：$2+\sqrt{2};2-\sqrt{2}$

解析：$f(x)=\dfrac{1-\cos 2x}{2}+\sin 2x+3\cdot\dfrac{1+\cos 2x}{2}$ (降階)

$$=\sin 2x+\cos 2x+2$$

$$=\sqrt{2}(\frac{1}{\sqrt{2}}\sin 2x+\frac{1}{\sqrt{2}}\cos 2x)+2$$

$$=\sqrt{2}(\sin 2x\cos\frac{\pi}{4}+\cos 2x\sin\frac{\pi}{4})+2$$

$=\sqrt{2}\sin(2x+\frac{\pi}{4})+2$,

$\because -1\le\sin(2x+\frac{\pi}{4})\le 1\Rightarrow -\sqrt{2}\le\sqrt{2}\sin(2x+\frac{\pi}{4})\le\sqrt{2}$

$\Rightarrow 2-\sqrt{2}\le\sqrt{2}\sin(2x+\frac{\pi}{4})+2\le 2+\sqrt{2}$,

最大值$=2+\sqrt{2}$ ，最小值$=2-\sqrt{2}$.