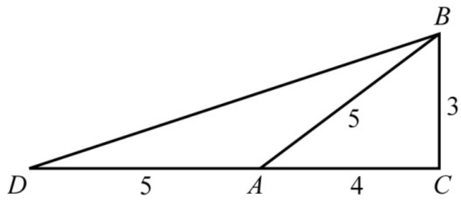
高雄市明誠中學 高三數學平時測驗 日期：100.11.10					
範圍	第 5 回三角函數	班級	三年　　班	姓名	
		座號			

一、填充題(每題 10 分)

1、 直角$\triangle ABC$中，$\angle C=90^\circ$, $\overline{AC}=4$, $\overline{BC}=3$，則$\tan\dfrac{A}{2}=$_______.

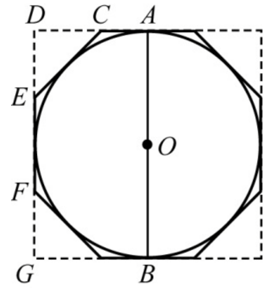


答案：$\dfrac{1}{3}$

解析：在$\overrightarrow{CA}$上取一點 D，使得$\overline{AD}=\overline{AB}=5$,

$\therefore \angle D=\dfrac{1}{2}\angle A$, 故$\tan\dfrac{A}{2}=\tan D=\dfrac{3}{5+4}=\dfrac{1}{3}$.

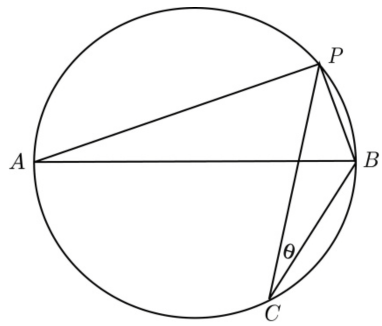
2、 邊長爲 1 的正八邊形其內接圓半徑爲______.



答案：$\dfrac{\sqrt{2}+1}{2}$

解析：$2r=\overline{AB}=\overline{DG}=\overline{DE}+\overline{EF}+\overline{FG}=\dfrac{\sqrt{2}}{2}+1+\dfrac{\sqrt{2}}{2}=\sqrt{2}+1$, $r=\dfrac{\sqrt{2}+1}{2}$.

3、 如圖,$\overline{AB}$爲直徑，且其長爲 26.若$\sin\theta=\dfrac{5}{13}$, 求$\overline{PA}+\overline{PB}=$______.



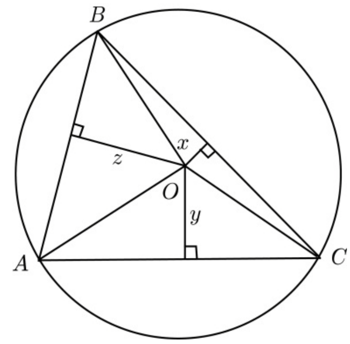
答案：34

解析：$\because \angle A=\dfrac{1}{2}\widehat{PB}=\angle\theta$, 又$\overline{AB}$爲直徑$\Rightarrow \angle APB=90^\circ$,

直角$\triangle APB$中，$\overline{PB}=\overline{AB}\cdot\sin\theta=26\cdot\dfrac{5}{13}=10$,

$\overline{PA}=\sqrt{26^2-10^2}=24$, $\therefore \overline{PA}+\overline{PB}=34$.

4、 若$\triangle ABC$中,$\angle A=75^\circ$,$\angle B=45^\circ$,O爲$\triangle ABC$之外接圓的圓心,且x,y,z分別表O到三邊$\overline{BC},\overline{AC},\overline{AB}$之垂線長，則$x:y:z=$______.



答案：$(\sqrt{6}-\sqrt{2}):2\sqrt{2}:2\sqrt{3}$

解析：$\because O$爲$\triangle ABC$之外心,

$\therefore \angle BOC=2\angle BAC=150^\circ$,

$\angle AOC=2\angle ABC=90^\circ$,

$\angle AOB=2\angle BCA=120^\circ$,

設$\overline{OA}=\overline{OB}=\overline{OC}=R$, $x=R\cos 75^\circ$, $y=R\cos 45^\circ$, $z=R\cos 60^\circ$,

$\therefore x:y:z=\cos 75^\circ:\cos 45^\circ:\cos 60^\circ=\dfrac{\sqrt{6}-\sqrt{2}}{4}:\dfrac{\sqrt{2}}{2}:\dfrac{\sqrt{3}}{2}=(\sqrt{6}-\sqrt{2}):2\sqrt{2}:2\sqrt{3}$.

5、 設θ爲銳角，且$\cos\theta=\sin^2\theta$，則$\dfrac{1}{1+\cos\theta}+\dfrac{1}{1-\cos\theta}=$______.

答案：$\sqrt{5}+1$

解析：$\cos\theta=\sin^2\theta=1-\cos^2\theta\Rightarrow\cos^2\theta+\cos\theta-1=0$

$\Rightarrow \cos\theta = \dfrac{-1+\sqrt{5}}{2} = \dfrac{\sqrt{5}-1}{2} > 0$ ($\dfrac{-1-\sqrt{5}}{2}$不合),

所求$= \dfrac{(1-\cos\theta)+(1+\cos\theta)}{1-\cos^2\theta} = \dfrac{2}{\sin^2\theta} = \dfrac{2}{\cos\theta} = \dfrac{4}{\sqrt{5}-1} = \sqrt{5}+1.$

6、 $\dfrac{1}{1+\sin^4\theta} + \dfrac{1}{1+\cos^4\theta} + \dfrac{1}{1+\tan^4\theta} + \dfrac{1}{1+\cot^4\theta} + \dfrac{1}{1+\sec^4\theta} + \dfrac{1}{1+\csc^4\theta} =$______.

答案：3

解析：$\because \dfrac{1}{1+\sin^4\theta} + \dfrac{1}{1+\csc^4\theta} = \dfrac{1}{1+\sin^4\theta} + \dfrac{1}{1+\dfrac{1}{\sin^4\theta}} = \dfrac{1}{1+\sin^4\theta} + \dfrac{\sin^4\theta}{1+\sin^4\theta} = 1,$

同理$\dfrac{1}{1+\cos^4\theta} + \dfrac{1}{1+\sec^4\theta} = 1$， $\dfrac{1}{1+\tan^4\theta} + \dfrac{1}{1+\cot^4\theta} = 1,$ $\therefore$所求$=1+1+1=3.$

7、 若$5\sin^2\theta + 3 = 10\sin\theta\cdot\cos\theta$，則$\tan\theta =$______.

答案：$\dfrac{3}{4}, \dfrac{1}{2}$

解析：同除以$\cos^2\theta$得$5\dfrac{\sin^2\theta}{\cos^2\theta} + \dfrac{3}{\cos^2\theta} = 10\dfrac{\sin\theta}{\cos\theta}$

$\Rightarrow 5\tan^2\theta + 3\sec^2\theta = 10\tan\theta$

$\Rightarrow 5\tan^2\theta + 3(1+\tan^2\theta) - 10\tan\theta = 0$

$\Rightarrow 8\tan^2\theta - 10\tan\theta + 3 = 0 \Rightarrow (4\tan\theta-3)(2\tan\theta-1)=0$ $\therefore \tan\theta = \dfrac{3}{4}, \dfrac{1}{2}.$

8、 若$x^2 - (\tan\theta+\cot\theta)x + 1 = 0$有一根爲$2-\sqrt{3}$，且$0<\theta<90^\circ$，則

(1) $\sin\theta\cdot\cos\theta =$______；(2) $\sin\theta+\cos\theta =$______；(3) $\sin^3\theta+\cos^3\theta =$______.

答案：(1) $\dfrac{1}{4}$ (2) $\dfrac{\sqrt{6}}{2}$ (3) $\dfrac{3\sqrt{6}}{8}$

解析：(1)設$\alpha \in \mathbb{R}$亦爲其根，

$\alpha(2-\sqrt{3}) = 1 \Rightarrow \alpha = \dfrac{1}{2-\sqrt{3}} = 2+\sqrt{3},$

$\therefore (2+\sqrt{3})+(2-\sqrt{3}) = \tan\theta+\cot\theta \Rightarrow 4 = \dfrac{\sin\theta}{\cos\theta} + \dfrac{\cos\theta}{\sin\theta} = \dfrac{\sin^2\theta+\cos^2\theta}{\sin\theta\cos\theta} = \dfrac{1}{\sin\theta\cos\theta},$

$\therefore \sin\theta\cos\theta = \dfrac{1}{4}.$

(2) $(\sin\theta+\cos\theta)^2 = \sin^2\theta + 2\sin\theta\cos\theta + \cos^2\theta = 1+2\cdot\dfrac{1}{4} = \dfrac{3}{2},$ $\therefore \sin\theta+\cos\theta = \sqrt{\dfrac{3}{2}} = \dfrac{\sqrt{6}}{2}.$

(3) $\sin^3\theta+\cos^3\theta = (\sin\theta+\cos\theta)^3 - 3\sin\theta\cos\theta(\sin\theta+\cos\theta)$

$= (\dfrac{\sqrt{6}}{2})^3 - 3\cdot\dfrac{1}{4}\cdot\dfrac{\sqrt{6}}{2} = \dfrac{6\sqrt{6}}{8} - \dfrac{3\sqrt{6}}{8} = \dfrac{3\sqrt{6}}{8}.$

9、 設$\tan\theta = \cos\theta$，則：(1) $\sin\theta =$______；(2) $\log_5(\dfrac{1}{1+\sin\theta} + \dfrac{1}{1-\sin\theta} - 1) =$______.

答案：(1) $\frac{\sqrt{5}-1}{2}$　(2) $\frac{1}{2}$

解析：(1) $\tan\theta=\cos\theta \Rightarrow \frac{\sin\theta}{\cos\theta}=\cos\theta \Rightarrow \sin\theta=\cos^2\theta=1-\sin^2\theta$

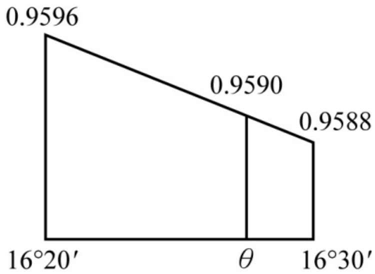
$\Rightarrow \sin^2\theta+\sin\theta-1=0 \Rightarrow \sin\theta=\frac{-1\pm\sqrt{1+4}}{2}=\frac{-1+\sqrt{5}}{2}$ ($\frac{-1-\sqrt{5}}{2}$不合).

(2) $\frac{1}{1+\sin\theta}+\frac{1}{1-\sin\theta}-1=\frac{1-\sin\theta+1+\sin\theta}{(1+\sin\theta)(1-\sin\theta)}-1=\frac{2}{1-\sin^2\theta}-1=\frac{2}{\cos^2\theta}-1=\frac{2}{\sin\theta}-1$

$=\frac{2}{\frac{\sqrt{5}-1}{2}}-1=\frac{4}{\sqrt{5}-1}-1=\sqrt{5}+1-1=\sqrt{5}$,

$\therefore$所求$=\log_5\sqrt{5}=\frac{1}{2}$.

10、若θ爲銳角，且 $\cos16^\circ20'=0.9596,\cos16^\circ30'=0.9588$，且 $\sin\theta=0.9590$，則$\theta=$______.

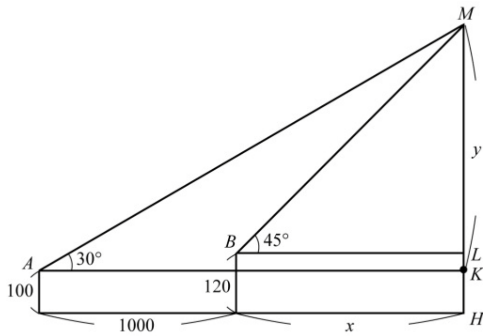


答案：$16^\circ27.5'$

解析：$\frac{\theta-16^\circ20'}{16^\circ30'-16^\circ20'}=\frac{0.9590-0.9596}{0.9588-0.9596}$

$\Rightarrow \theta=16^\circ20'+10'\cdot\frac{-0.0006}{-0.0008}=16^\circ20'+7.5'=16^\circ27.5'$.

11、在海拔 100 m 的 A 點測得山頂上 M 點的仰角爲30°，由 M 水平向前 1 公里後，在海拔 120 m 的 B 點測得山頂 M 的仰角爲 45°，試求山高爲______公尺.



答案：$590+490\sqrt{3}$

解析：設$\overline{BL}=x$,$\overline{MK}=y$，$\overline{AK}=x+1000$

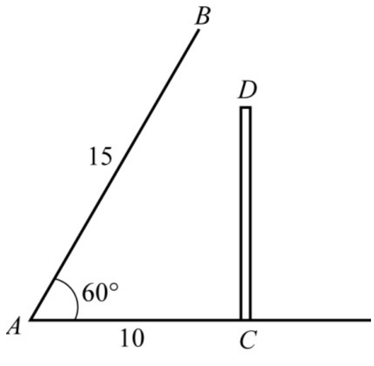
$$\begin{cases}\frac{y}{x+1000}=\frac{1}{\sqrt{3}} & (in\ \triangle AMK)\\ y-20=x & (in\ \triangle BML)\end{cases}$$

$\Rightarrow \frac{y}{y+980}=\frac{1}{\sqrt{3}} \Rightarrow \sqrt{3}y=y+980 \Rightarrow (\sqrt{3}-1)y=980$

$\Rightarrow y=\frac{980}{\sqrt{3}-1}=\frac{980(\sqrt{3}+1)}{(\sqrt{3}-1)(\sqrt{3}+1)}=\frac{980(\sqrt{3}+1)}{2}=490(\sqrt{3}+1)$,

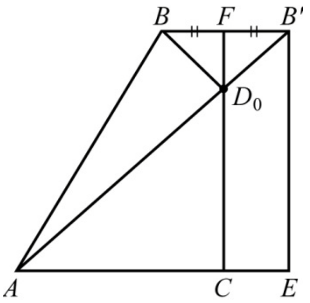
$\overline{MH}=y+100=590+490\sqrt{3}$.

12、有一長形物體$\overline{AB}$立於地面上，且與地面保持60°的傾斜角.如圖，設在距 A 點 10 公尺處垂直置放一面鏡子$\overline{CD}$，已知$\overline{AB}=15$公尺.設在 A 處觀察，則鏡子$\overline{CD}$至少需______公尺高才能在鏡中看到整個物體$\overline{AB}$，設此時的鏡子以$\overline{CD_0}$表示，則$\overline{AD_0}+\overline{BD_0}$之值爲______.



答案：$6\sqrt{3}$; $5\sqrt{13}$

解析：



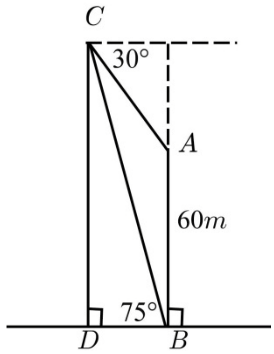
如圖，$\overline{B'F}=\overline{BF}=\overline{AC}-\overline{AB}\cos 60^\circ=10-15\cdot\frac{1}{2}=\frac{5}{2}$

$\overline{B'E}=\overline{AB}\cdot\sin 60^\circ=15\cdot\frac{\sqrt{3}}{2}=\frac{15\sqrt{3}}{2}$

又 $\frac{\overline{CD_0}}{\overline{B'E}}=\frac{\overline{AC}}{\overline{AE}}\Rightarrow\overline{CD_0}=\frac{10}{10+\frac{5}{2}}\cdot\frac{15\sqrt{3}}{2}=6\sqrt{3}$

$\overline{AD_0}+\overline{BD_0}=\overline{AD_0}+\overline{B'D_0}=\overline{AB'}=\sqrt{\overline{AE}^2+\overline{B'E}^2}=\sqrt{(10+\frac{5}{2})^2+(\frac{15\sqrt{3}}{2})^2}=5\sqrt{13}$.

13、如圖，小明在他家的樓底 B 點測得焚化爐的頂樓 C 點之仰角爲 $75°$,他去焚化爐頂樓參觀時，看到他家樓頂 A 點的俯角是 $30°$,若 $\overline{AB}=60$ m，則焚化爐的高度爲________m.



答案：$15(3+\sqrt{3})$

解析：$\angle ACB=75°-30°=45°,\angle ABC=15°\Rightarrow\angle BAC=120°$

$\triangle ABC$ 中，$\frac{60}{\sin 45°}=\frac{\overline{BC}}{\sin 120°}\Rightarrow\overline{BC}=60\sqrt{2}\cdot\frac{\sqrt{3}}{2}=30\sqrt{6}$,

$\therefore\overline{CD}=30\sqrt{6}\cdot\sin 75°=30\sqrt{6}\cdot\frac{\sqrt{6}+\sqrt{2}}{4}=\frac{30(6+2\sqrt{3})}{4}=15(3+\sqrt{3})$ m.

14、$\cos(-10^\circ)+\cos 80^\circ+\cos 100^\circ+\cos 170^\circ+\cos 260^\circ+\cos 280^\circ=$________.

答案：0

解析：原式= $\cancel{\cos 10^\circ}+\cancel{\cos 80^\circ}-\cancel{\cos 80^\circ}-\cancel{\cos 10^\circ}-\cos 80^\circ+\cos 80^\circ=0$.

15、$\log_8(\frac{\cos 30^\circ+\sin 150^\circ}{\cos 330^\circ+\sin 210^\circ})-2\log_8(1+\tan 240^\circ)=$________.

答案：$-\frac{1}{3}$

解析：原式$=\log_8\left(\frac{\frac{\sqrt{3}}{2}+\frac{1}{2}}{\frac{\sqrt{3}}{2}-\frac{1}{2}}\right)-\log_8(1+\sqrt{3})^2=\log_8(\frac{\sqrt{3}+1}{\sqrt{3}-1}\cdot\frac{1}{(\sqrt{3}+1)^2})$

$=\log_8(\frac{1}{(\sqrt{3}-1)(\sqrt{3}+1)})=\log_8\frac{1}{2}=-\frac{1}{3}$.

16、若 $\sin(630^\circ-\theta^\circ)=\frac{1}{2}$，且 θ 在第三象限，則 $\cos(\theta-1260^\circ)=$________.

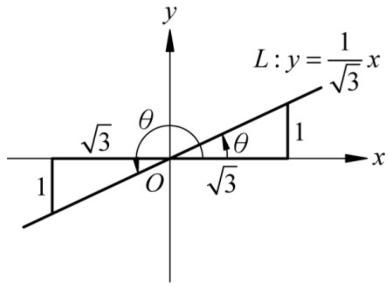
答案：$\frac{1}{2}$

解析：$\sin(630^\circ-\theta)=\sin(90^\circ\times 7-\theta)=-\cos\theta\Rightarrow\cos\theta=-\frac{1}{2}$,

$\cos(\theta-1260^\circ)=\cos(1260^\circ-\theta)=\cos(90^\circ\times 14-\theta)=-\cos\theta=\frac{1}{2}$.

17、若θ角的終邊落在直線$L: y=\dfrac{1}{\sqrt{3}}x$上，則$\sin\theta=$________, $\cos\theta=$________.

答案：$\pm\dfrac{1}{2};\pm\dfrac{\sqrt{3}}{2}$



解析： (1)取 $y=1$時，$x=\sqrt{3}$,

此時$r=\sqrt{1^2+(\sqrt{3})^2}=2\Rightarrow$故$\sin\theta=\dfrac{1}{2}$，$\cos\theta=\dfrac{\sqrt{3}}{2}$.

(2) $y=-1$時，$x=-\sqrt{3}$,

此時$r=\sqrt{1^2+(-\sqrt{3})^2}=2\Rightarrow$故$\sin\theta=-\dfrac{1}{2}$，$\cos\theta=-\dfrac{\sqrt{3}}{2}$.

18、若$\sin\theta+\cos\theta=\dfrac{\sqrt{3}-1}{2}$，且$0<\theta<180^\circ$，則$\tan\theta=$________.

答案：$-\sqrt{3}$

解析：$\because\begin{cases}\sin\theta+\cos\theta=\dfrac{\sqrt{3}-1}{2}\\ \sin^2\theta+\cos^2\theta=1\end{cases}$,

$$(\sin\theta+\cos\theta)^2=\sin^2\theta+2\sin\theta\cos\theta+\cos^2\theta\Rightarrow(\frac{\sqrt{3}-1}{2})^2=1+2\sin\theta\cos\theta$$

$$\Rightarrow\frac{4-2\sqrt{3}}{4}=1+2\sin\theta\cos\theta\Rightarrow 2\sin\theta\cos\theta=\frac{-\sqrt{3}}{2},$$

$$\sin^2\theta+\cos^2\theta-2\sin\theta\cos\theta=1+\frac{\sqrt{3}}{2}\Rightarrow(\sin\theta-\cos\theta)^2=\frac{2+\sqrt{3}}{2}$$

$$\Rightarrow\sin\theta-\cos\theta=\sqrt{\frac{4+2\sqrt{3}}{4}}=\frac{\sqrt{3}+1}{2},$$

$$\therefore\begin{cases}\sin\theta+\cos\theta=\dfrac{\sqrt{3}-1}{2}\\ \sin\theta-\cos\theta=\dfrac{\sqrt{3}+1}{2}\end{cases},\ \text{得}\sin\theta=\frac{\sqrt{3}}{2},\cos\theta=-\frac{1}{2}\quad\text{，故}\tan\theta=\frac{\frac{\sqrt{3}}{2}}{\frac{-1}{2}}=-\sqrt{3}.$$

19、若$2\sin\theta+\cos\theta=0$,則$2\sin^2\theta-3\sin\theta\cos\theta-5\cos^2\theta=$________.

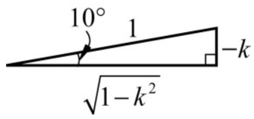
答案：$-\dfrac{12}{5}$

解析：$2\sin\theta=-\cos\theta\Rightarrow\dfrac{\sin\theta}{\cos\theta}=-\dfrac{1}{2}=\tan\theta$,

$$\text{所求}=\frac{2\sin^2\theta-3\sin\theta\cos\theta-5\cos^2\theta}{1}=\frac{2\sin^2\theta-3\sin\theta\cos\theta-5\cos^2\theta}{\sin^2\theta+\cos^2\theta}$$

$$=\frac{2\dfrac{\sin^2\theta}{\cos^2\theta}-3\dfrac{\sin\theta}{\cos\theta}-5}{\dfrac{\sin^2\theta}{\cos^2\theta}+1}=\frac{2\tan^2\theta-3\tan\theta-5}{\tan^2\theta+1}=\frac{2\cdot\frac{1}{4}-3(-\frac{1}{2})-5}{\frac{1}{4}+1}=\frac{\frac{1}{2}+\frac{3}{2}-5}{\frac{5}{4}}=\frac{-3}{\frac{5}{4}}=\frac{-12}{5}.$$

20、設$\cos(-100^\circ)=k$，則$\tan190^\circ=$________.



答案：$\dfrac{-k\sqrt{1-k^2}}{1-k^2}$

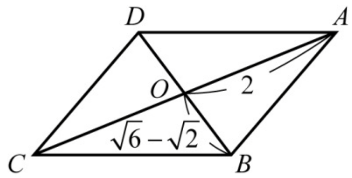
解析：$\cos(-100^\circ)=\cos100^\circ=\cos(90^\circ+10^\circ)=-\sin10^\circ=k<0$,

$\therefore \sin10^\circ=-k=\dfrac{-k}{1}>0$

$\tan190^\circ=\tan(90^\circ\times2+10^\circ)=\tan10^\circ=\dfrac{-k}{\sqrt{1-k^2}}=\dfrac{-k\sqrt{1-k^2}}{1-k^2}$.

21、平行四邊形$ABCD$的對角線交點爲O，對角線$\overline{AC}=4$, $\overline{BD}=2\sqrt6-2\sqrt2$, $\angle COD=45^\circ$，則$\overline{AB}=$________, $\overline{BC}=$________, $\angle DBC=$________.

答案：$2\sqrt3-2;2\sqrt2;30^\circ$



解析：$\triangle OAB$中，

$\overline{AB}^2=2^2+(\sqrt6-\sqrt2)^2-2\cdot2\cdot(\sqrt6-\sqrt2)\cos45^\circ$

$=4+(8-4\sqrt3)-4(\sqrt6-\sqrt2)\cdot\dfrac{1}{\sqrt2}$

$=12-4\sqrt3-4(\sqrt3-1)=16-8\sqrt3$,

$\therefore \overline{AB}=\sqrt{16-8\sqrt3}=2\sqrt{4-2\sqrt3}=2(\sqrt3-1)=2\sqrt3-2$.

$\triangle OBC$中，$\overline{BC}^2=2^2+(\sqrt6-\sqrt2)^2-2\cdot2(\sqrt6-\sqrt2)\cos135^\circ$

$=4+(8-4\sqrt3)-4(\sqrt6-\sqrt2)(\dfrac{-1}{\sqrt2})$

$=12-4\sqrt3+4(\sqrt3-1)=8$,

$\therefore \overline{BC}=\sqrt8=2\sqrt2$.

$\triangle OBC$中，$2^2=(\sqrt6-\sqrt2)^2+(2\sqrt2)^2-2(2\sqrt2)(\sqrt6-\sqrt2)\cos\angle OBC$

$\Rightarrow 4=(8-4\sqrt3)+8-8(\sqrt3-1)\cos\angle DBC$

$\Rightarrow 8(\sqrt3-1)\cos\angle DBC=12-4\sqrt3$

$\Rightarrow \cos\angle DBC=\dfrac{3-\sqrt3}{2(\sqrt3-1)}=\dfrac{\sqrt3}{2}$, $\quad\therefore \angle DBC=30^\circ$.

22、設$\triangle ABC$之三邊長爲$a,b,\sqrt{a^2+ab+b^2}$，則$\triangle ABC$之最大內角爲________度.

答案：120

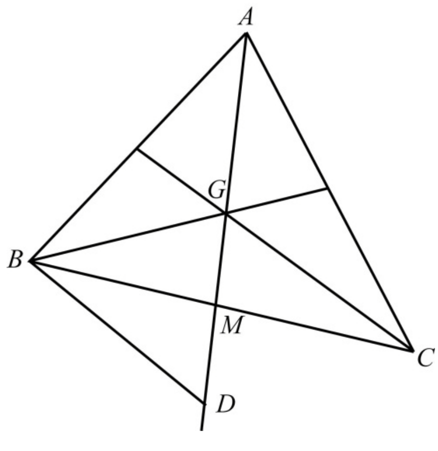
解析：$\because \sqrt{a^2+ab+b^2}>\sqrt{a^2}=a$ 且 $\sqrt{a^2+ab+b^2}>\sqrt{b^2}=b$, $\quad\therefore$最大邊爲$\sqrt{a^2+ab+b^2}$，

$a^2+ab+b^2=a^2+b^2-2ab\cos\theta\Rightarrow\cos\theta=-\dfrac12\Rightarrow\theta=120^\circ$.

23、$\triangle ABC$三中線長爲 7,8,9，則$\triangle ABC$之面積爲________.

答案：$16\sqrt5$

解析：取$\overline{BC}$之中點M，$\triangle ABC$之重心G，在$\overrightarrow{AM}$上取一點D，



使 $\overline{GM}=\overline{MD}$，$\because \overline{GM}=\overline{MD}, \overline{BM}=\overline{MC}, \angle GMC=\angle BMD,$

$\therefore \triangle GMC \cong \triangle DMB$ (SAS),

$\overline{GD}=\overline{GA}=\frac{2}{3}\cdot 7=\frac{14}{3}, \quad \overline{BG}=\frac{2}{3}\cdot 8=\frac{16}{3}, \quad \overline{CG}=\overline{BD}=\frac{2}{3}\cdot 9=6,$

$\triangle BGD$ 中，$s=\frac{1}{2}(\frac{14}{3}+\frac{16}{3}+6)=8,$

$\triangle BGD=\sqrt{8\cdot(8-\frac{14}{3})(8-\frac{16}{3})(8-6)}=\sqrt{8\cdot\frac{10}{3}\cdot\frac{8}{3}\cdot 2}=\frac{16}{3}\sqrt{5},$

$\triangle ABC=3\triangle BGD=16\sqrt{5}$.

24、設 $\triangle ABC$ 之三高分別爲 $h_a=\frac{3\sqrt{15}}{4}, h_b=\frac{3\sqrt{15}}{2}, h_c=\sqrt{15}$，則

(1)最小角之餘弦爲________；(2) $\triangle ABC$ 之面積爲________；(3) $\triangle ABC$ 之周長爲________.

答案：(1) $\frac{7}{8}$ (2) $3\sqrt{15}$ (3)18

解析：(1) $\triangle ABC=\frac{1}{2}a\cdot\frac{3\sqrt{15}}{4}=\frac{1}{2}b\cdot\frac{3\sqrt{15}}{2}=\frac{1}{2}c\sqrt{15},$

$a:b:c=\frac{1}{h_a}:\frac{1}{h_b}:\frac{1}{h_c}\Rightarrow a:b:c=\frac{4}{3}:\frac{2}{3}:1=4:2:3,$ 最小角爲 $\angle B$

$\cos B=\frac{4^2+3^2-2^2}{2\cdot 4\cdot 3}\Rightarrow \cos B=\frac{7}{8}.$

(2) $\cos B=\frac{7}{8}\Rightarrow \sin B=\frac{\sqrt{15}}{8}$，又 $\sin B=\frac{h_a}{c}\Rightarrow \frac{h_a}{c}=\frac{\sqrt{15}}{8}, \quad c=\frac{3\sqrt{15}}{4}\times\frac{8}{\sqrt{15}}=6$

$\triangle ABC=\frac{1}{2}c\sqrt{15}=\frac{1}{2}\times 6\times\sqrt{15}=3\sqrt{15}$

※另解 $a=\frac{8\Delta}{3\sqrt{15}}, b=\frac{4\Delta}{3\sqrt{15}}, c=\frac{2\Delta}{\sqrt{15}}$, 其中 Δ 爲 $\triangle ABC$ 面積

$s=\frac{1}{2}(\frac{8\Delta}{3\sqrt{15}}+\frac{4\Delta}{3\sqrt{15}}+\frac{2\Delta}{\sqrt{15}})=\frac{1}{2}\cdot\frac{18\Delta}{3\sqrt{15}}=\frac{3\Delta}{\sqrt{15}},$

$\therefore \Delta=\sqrt{\frac{3\Delta}{\sqrt{15}}(\frac{3\Delta}{\sqrt{15}}-\frac{8\Delta}{3\sqrt{15}})(\frac{3\Delta}{\sqrt{15}}-\frac{4\Delta}{3\sqrt{15}})(\frac{3\Delta}{\sqrt{15}}-\frac{2\Delta}{\sqrt{15}})}=\sqrt{\frac{3\Delta}{\sqrt{15}}\cdot\frac{\Delta}{3\sqrt{15}}\cdot\frac{5\Delta}{3\sqrt{15}}\cdot\frac{\Delta}{\sqrt{15}}}$

$=\frac{\Delta^2\cdot\sqrt{15}}{45},$

$\Rightarrow \Delta=\frac{45}{\sqrt{15}}=3\sqrt{15}.$

(3)周長 $=2s=\frac{6\cdot 3\sqrt{15}}{\sqrt{15}}=18.$

25、$\triangle ABC$ 中，若 $\angle A, \angle B, \angle C$ 之對應邊分別爲 a,b,c，且 $(b+c):(c+a):(a+b)=4:5:6$，則

(1) $\sin A:\sin B:\sin C=$________；(2) $\cos A:\cos B:\cos C=$________；(3) $\sin A=$________.

答案：(1)$7:5:3$　(2)$(-7):11:13$　(3)$\frac{\sqrt{3}}{2}$

解析：(1)設$\frac{b+c}{4}=\frac{c+a}{5}=\frac{a+b}{6}=k, k\neq 0$，$\begin{cases} b+c=4k \\ c+a=5k \\ a+b=6k \end{cases}$

$\Rightarrow 2(a+b+c)=15k \Rightarrow a+b+c=\frac{15k}{2}$,　$\therefore a=\frac{7k}{2}, b=\frac{5k}{2}, c=\frac{3k}{2}$,

故$\sin A:\sin B:\sin C=a:b:c=7:5:3$.

(2)$\cos A:\cos B:\cos C=\frac{b^2+c^2-a^2}{2bc}:\frac{a^2+c^2-b^2}{2ac}:\frac{a^2+b^2-c^2}{2ab}$

$=a(b^2+c^2-a^2):b(a^2+c^2-b^2):c(a^2+b^2-c^2)$

$=7(25+9-49):5(49+9-25):3(49+25-9)$

$=7\times(-15):(5\times 33):(3\times 65)=(-7):11:13$.

(3)$\cos A=\frac{25+9-49}{2\cdot 5\cdot 3}=\frac{-15}{30}=-\frac{1}{2}$,　$\therefore \angle A=120^\circ \Rightarrow \sin A=\frac{\sqrt{3}}{2}$.

26、$\triangle ABC$中，若$\angle A, \angle B, \angle C$之對應邊分別為$a,b,c$，且$\frac{5}{12}(a-b+c)=\sin A-\sin B+\sin C$，則$\triangle ABC$之外接圓半徑為________.

答案：$\frac{6}{5}$

解析：$\sin A-\sin B+\sin C=\frac{a}{2R}-\frac{b}{2R}+\frac{c}{2R}=\frac{1}{2R}(a-b+c)$,

$\therefore \frac{1}{2R}=\frac{5}{12}\Rightarrow R=\frac{6}{5}$.

27、$\triangle ABC$之三中線長為$m_a=\frac{\sqrt{145}}{2}, m_b=\frac{\sqrt{73}}{2}, m_c=2\sqrt{7}$，則$a=$______$,b=$______$,c=$______.

答案：5; 7; 6

解析：$m_a=\frac{1}{2}\sqrt{2(b^2+c^2)-a^2}=\frac{\sqrt{145}}{2}\Rightarrow 2b^2+2c^2-a^2=145\cdots\cdots$①

$m_b=\frac{1}{2}\sqrt{2(a^2+c^2)-b^2}=\frac{\sqrt{73}}{2}\Rightarrow 2a^2+2c^2-b^2=73\cdots\cdots$②

$m_c=\frac{1}{2}\sqrt{2(a^2+b^2)-c^2}=2\sqrt{7}\Rightarrow 2a^2+2b^2-c^2=112\cdots\cdots$③

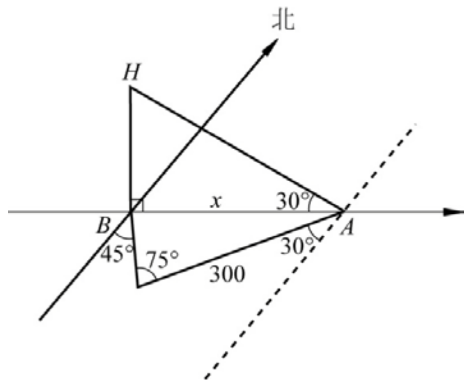
①+②+③ 得$3(a^2+b^2+c^2)=330$　$\Rightarrow a^2+b^2+c^2=110$,

$\begin{cases} 2(110-a^2)-a^2=145 \\ 2(110-b^2)-b^2=73 \\ 2(110-c^2)-c^2=112 \end{cases}\Rightarrow \begin{cases} 3a^2=75 \\ 3b^2=147 \\ 3c^2=108 \end{cases}\Rightarrow \begin{cases} a^2=25 \\ b^2=49 \\ c^2=36 \end{cases}$,　$\therefore a=5, b=7, c=6$.

28、平面上有 A、B 兩點，A 在塔的正東，B 在塔的東南且在 A 的南 30° 西 300 公尺處，在 A 測得塔頂的仰角爲 30° ，則塔高爲________公尺.

答案：$50(3+\sqrt{3})$

解析：$\because \dfrac{x}{\sin 75^\circ}=\dfrac{300}{\sin 45^\circ}\Rightarrow x=\dfrac{300}{\sin 45^\circ}\cdot\sin 75^\circ$,



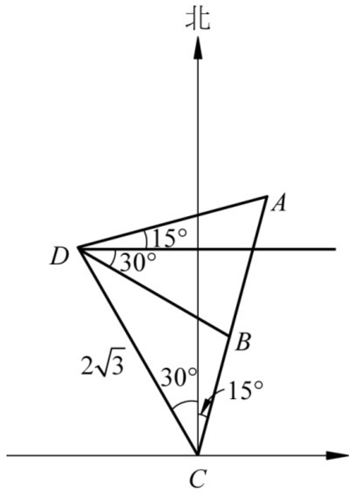
即 $x=\dfrac{300}{\frac{1}{\sqrt{2}}}\cdot\dfrac{\sqrt{6}+\sqrt{2}}{4}=\dfrac{300\cdot(2+2\sqrt{3})}{4}=150(\sqrt{3}+1)$,

故 $\overline{OH}=\dfrac{150(\sqrt{3}+1)}{\sqrt{3}}=50(3+\sqrt{3})$.

29、某船位於 C 看見 A、B 二燈塔在北偏東 15° 之方向上成一直線，船向北 30° 西方向航行 $2\sqrt{3}$ 哩到達 D，此時看見 A 燈塔在東 15° 北的方向上，另一燈塔 B 在東 30° 南的方向上，則 A、B 兩燈塔之距離爲________哩，又 D 與燈塔 B 之距離爲________哩.

答案：$2(\sqrt{6}-\sqrt{2});6-2\sqrt{3}$

解析：$\triangle ACD$ 中，$\dfrac{\overline{AD}}{\sin 45^\circ}=\dfrac{2\sqrt{3}}{\sin 60^\circ}\Rightarrow \overline{AD}=\dfrac{2\sqrt{3}}{\frac{\sqrt{3}}{2}}\cdot\dfrac{1}{\sqrt{2}}=2\sqrt{2}$,



$\triangle ABD$ 中，$\dfrac{\overline{AB}}{\sin 45^\circ}=\dfrac{2\sqrt{2}}{\sin 75^\circ}=\dfrac{\overline{BD}}{\sin 60^\circ}$.

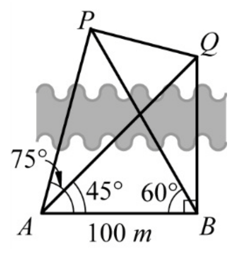
$\therefore \overline{AB}=\dfrac{2\sqrt{2}}{\frac{\sqrt{6}+\sqrt{2}}{4}}\cdot\dfrac{1}{\sqrt{2}}=\dfrac{8}{\sqrt{6}+\sqrt{2}}=2(\sqrt{6}-\sqrt{2})$,

$\overline{BD}=\dfrac{2\sqrt{2}}{\frac{\sqrt{6}+\sqrt{2}}{4}}\cdot\dfrac{\sqrt{3}}{2}=\dfrac{4\sqrt{6}}{\sqrt{6}+\sqrt{2}}=\sqrt{6}(\sqrt{6}-\sqrt{2})=6-2\sqrt{3}$.

30、設一湖，欲測湖岸兩點 P、Q 的距離，已知湖岸築有鐵絲網不能靠近，今在鐵絲網外取得 A、B 兩點，得 $\overline{AB}=100$ 公尺，如圖，測得 $\angle PAB=75^\circ,\angle QAB=45^\circ,\angle PBA=60^\circ,\angle QBA=90^\circ$ ，則 $\overline{AP}=$________公尺；$\overline{PQ}=$________公尺.

答案：$50\sqrt{6}$ ；$50\sqrt{2}$

解析：(1) $\triangle PAB$ 中，$\angle APB=180^\circ-(75^\circ+60^\circ)=45^\circ$,



由正弦定理：$\dfrac{\overline{AP}}{\sin 60^\circ}=\dfrac{100}{\sin 45^\circ}\Rightarrow \overline{AP}=50\sqrt{6}$,

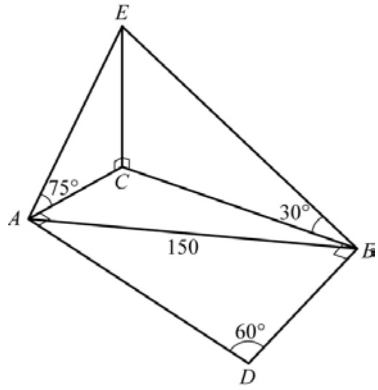
(2) $\triangle QAB$ 中，$\angle ABQ=90^\circ$, $\therefore \overline{AQ}=100\sqrt{2}$,

又 $\angle PAQ=75^\circ-45^\circ=30^\circ$,

由餘弦定理：

$\overline{PQ}^2=(50\sqrt{6})^2+(100\sqrt{2})^2-2\cdot 50\sqrt{6}\cdot 100\sqrt{2}\cos 30^\circ=5000$, $\therefore \overline{PQ}=\sqrt{5000}=50\sqrt{2}$.

31、由一河兩岸的 A、B 兩點，同時測一飛過地面上 C 點上方天空的氣球，分別測得仰角是 75°，30°，又由地面另一點 D，測得 $\angle CAD$ 與 $\angle CBD$ 都是直角，而 $\angle ADB=60^\circ$ (A 與 C 在河之同側，B 與 D 同在河的另一側)，若已知河寬 AB 是 150 公尺，則氣球當時的高度=______公尺.



答案：$\dfrac{150}{\sqrt{7-2\sqrt{3}}}$

解析：$\because \angle ADB=60^\circ$,且 $\angle DAL$ 和 $\angle DBC=90^\circ$， $\therefore \angle ACB=120^\circ$,

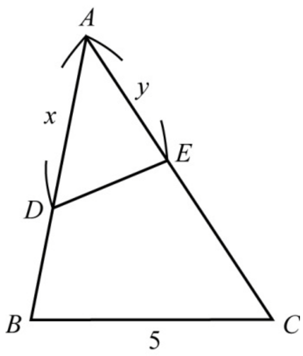
設 $\overline{CE}=h$ ，則 $\overline{AC}=(2-\sqrt{3})h, \overline{BC}=\sqrt{3}h$ ，

由餘弦定理知，

$150^2=[(2-\sqrt{3})h]^2+(\sqrt{3}h)^2-2\cdot\sqrt{3}h\cdot(2-\sqrt{3})h\cdot\cos 120^\circ=(7-2\sqrt{3})h^2$,

$\therefore h=\dfrac{150}{\sqrt{7-2\sqrt{3}}}$.

32、 $\triangle ABC$ 中， $\overline{BC}=5, \overline{CA}=7, \overline{AB}=6$ ，在 $\overline{AB}, \overline{AC}$ 上分別取 D, E 兩點，使得 $\triangle ADE$ 的面積是 $\triangle ABC$ 的 $\dfrac{1}{3}$ ，則 $\overline{DE}$ 的最小值爲________.



答案：$2\sqrt{2}$

解析：設 $\overline{AD}=x, \overline{AE}=y$ ，

$5^2=6^2+7^2-2\cdot 6\cdot 7\cos A \Rightarrow 25=36+49-84\cos A \quad \Rightarrow \cos A=\dfrac{60}{84}=\dfrac{5}{7}$,

$\triangle ADE=\dfrac{1}{3} \quad ABC \Rightarrow \dfrac{1}{2}xy\sin A=\dfrac{1}{3}\cdot(\dfrac{1}{2}\cdot 6\cdot 7\sin A) \Rightarrow xy=14$,

又 $\overline{DE}^2=x^2+y^2-2xy\cos A=x^2+y^2-2\cdot 14\cdot\dfrac{5}{7}=x^2+y^2-20$,

且 $\dfrac{x^2+y^2}{2}\ge\sqrt{x^2y^2} \Rightarrow x^2+y^2\ge 2\sqrt{x^2y^2}=2xy$ ，即$\Rightarrow x^2+y^2\ge 28$,

$\overline{DE}^2=x^2+y^2-20\ge 28-20=8$,

$\overline{DE}\ge\sqrt{8}=2\sqrt{2}$, 故 $\overline{DE}$ 之最小值爲 $2\sqrt{2}$.