

高雄市明誠中學 高二數學平時測驗 日期：100.09.20				
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一、填充題 (每題 10 分)

1. $\triangle ABC$ 中, $\angle A = 45^\circ$, $\angle B = 60^\circ$, $\overline{BC} = 2$, 則: (1) $\overline{AB} =$ _____ . (2) $\overline{AC} =$ _____ .

解答 (1) $\sqrt{3}+1$; (2) $\sqrt{6}$

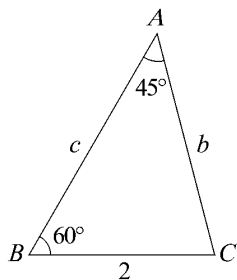
解析 (1) $\angle C = 180^\circ - 45^\circ - 60^\circ = 75^\circ$,

$$\text{由正弦定理: } \frac{a}{\sin A} = \frac{c}{\sin C} \Rightarrow \frac{2}{\sin 45^\circ} = \frac{c}{\sin 75^\circ}$$

$$\Rightarrow c \sin 45^\circ = 2 \sin 75^\circ \Rightarrow c = \frac{2 \sin 75^\circ}{\sin 45^\circ} = 2 \times \frac{\sqrt{6} + \sqrt{2}}{4} \times \frac{2}{\sqrt{2}} = \sqrt{3} + 1 .$$

$$(2) \text{由正弦定理: } \frac{a}{\sin A} = \frac{b}{\sin B} \Rightarrow \frac{2}{\sin 45^\circ} = \frac{b}{\sin 60^\circ}$$

$$\Rightarrow b \sin 45^\circ = 2 \sin 60^\circ \Rightarrow c = \frac{2 \sin 60^\circ}{\sin 45^\circ} = 2 \times \frac{\sqrt{3}}{2} \times \frac{2}{\sqrt{2}} = \sqrt{6} .$$



2. $\triangle ABC$ 中, $\overline{AB} = \sqrt{3}$, $\overline{AC} = 1$, $\angle B = 30^\circ$, 且 $\triangle ABC$ 不是直角三角形, 則:

(1) $\overline{BC} =$ _____ . (2) $\angle C =$ _____ .

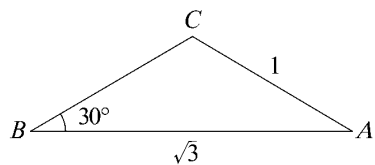
解答 (1) 1; (2) 120°

解析 (1) 由餘弦定理知, $b^2 = a^2 + c^2 - 2accosB$

$$\Rightarrow 1^2 = a^2 + (\sqrt{3})^2 - 2a \cdot \sqrt{3} \cos 30^\circ \Rightarrow a^2 - 3a + 2 = 0 \Rightarrow a = 1 \text{ 或 } 2,$$

$\because \triangle ABC$ 不是直角三角形, $\therefore a = 1$.

(2) $\triangle ABC$ 中, $\because \overline{AC} = \overline{BC} = 1$, $\angle A = \angle B = 30^\circ$, $\angle C = 180^\circ - 30^\circ - 30^\circ = 120^\circ$.



3. $\triangle ABC$ 中, $b = 4$, $c = 2$, $\tan B = \sqrt{15}$, 則 $a =$ _____ .

解答 4

解析 $\tan B = \frac{\sqrt{15}}{1} \Rightarrow \cos B = \frac{1}{4}$, $\therefore b^2 = c^2 + a^2 - 2cacosB$

$$\Rightarrow 16 = 4 + a^2 - 2 \cdot 2 \cdot a \cdot \frac{1}{4} \Rightarrow a^2 - a - 12 = 0 \Rightarrow a = 4 .$$

4. $\triangle ABC$ 中, $a = \sqrt{3} - 1$, $c = \sqrt{3} + 1$, $\angle A = 15^\circ$, 則 $b =$ _____ .

解答 $2\sqrt{2}$ 或 $\sqrt{6}$

解析 $(\sqrt{3} - 1)^2 = (\sqrt{3} + 1)^2 + b^2 - 2b(\sqrt{3} + 1)\cos 15^\circ$

$$\Rightarrow 4 - 2\sqrt{3} = 4 + 2\sqrt{3} + b^2 - 2b(\sqrt{3} + 1) \frac{\sqrt{6} + \sqrt{2}}{4} = 4 + 2\sqrt{3} + b^2 - \sqrt{2}(2 + \sqrt{3})b$$

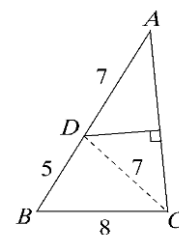
$$\Rightarrow b^2 - (2\sqrt{2} + \sqrt{6})b + 4\sqrt{3} = 0 \Rightarrow (b - 2\sqrt{2})(b - \sqrt{6}) = 0 \Rightarrow b = 2\sqrt{2} \text{ 或 } \sqrt{6} .$$

5. $\triangle ABC$ 中, 若 \overline{AC} 的中垂線交 \overline{AB} 於 D , 若 $\overline{AD} = 7$, $\overline{BD} = 5$, $\overline{BC} = 8$, 則 $\overline{AC} =$ _____ .

解答 $4\sqrt{7}$

解析 $\cos B = \frac{25 + 64 - 49}{2 \cdot 5 \cdot 8} = \frac{1}{2}$,

$$\therefore \overline{AC}^2 = 12^2 + 8^2 - 2 \cdot 12 \cdot 8 \cdot \frac{1}{2} = 112 \Rightarrow \overline{AC} = 4\sqrt{7} .$$



6. $\triangle ABC$ 之三邊長為 8, 10, 12, 則:

(1) $\triangle ABC$ 之面積為 _____ . (2) $\triangle ABC$ 之外接圓半徑為 _____ . (3) $\triangle ABC$ 最大邊上之中線長為 _____ .

解答 (1) $15\sqrt{7}$; (2) $\frac{16\sqrt{7}}{7}$; (3) $\sqrt{46}$

解析 (1) 設 $a = 8$, $b = 10$, $c = 12$, 則 $s = \frac{1}{2}(8 + 10 + 12) = 15$,

$$\text{由海龍公式 } \Delta = \sqrt{15 \cdot 3 \cdot 5 \cdot 7} = 15\sqrt{7} .$$

$$(2) \text{由 } \Delta = \frac{abc}{4R} \Rightarrow 15\sqrt{7} = \frac{8 \times 10 \times 12}{4R} \Rightarrow R = \frac{240}{15\sqrt{7}} = \frac{16\sqrt{7}}{7} .$$

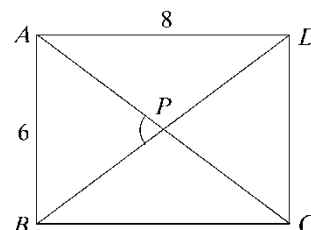
$$(3) \text{最大邊上之中線長} = \frac{1}{2} \sqrt{2a^2 + 2b^2 - c^2} = \frac{1}{2} \sqrt{2 \cdot 8^2 + 2 \cdot 10^2 - 12^2} = \sqrt{46} .$$

7. 長方形 $ABCD$, 令 $\overline{AB} = 6$, $\overline{AD} = 8$, 對角線 \overline{AC} 與 \overline{BD} 相交於 P 點, 求 $\cos \angle APB =$ _____ .

解答 $\frac{7}{25}$

解析 $\overline{AB} = 6$, $\overline{AD} = 8$, $\overline{AC} = \overline{BD} = 10$, $\overline{AP} = \overline{BP} = 5$

$$\Rightarrow \cos \angle APB = \frac{5^2 + 5^2 - 6^2}{2 \times 5 \times 5} = \frac{14}{2 \times 5 \times 5} = \frac{7}{25} .$$



8. $\triangle ABC$ 中, $\angle B = 45^\circ$, $\angle C = 60^\circ$, $a = \sqrt{3} + 1$, 求:

(1) \overline{AB} 的值 = _____ . (2) 外接圓半徑 = _____ .

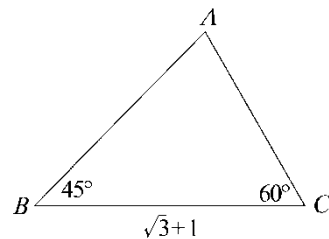
解答 (1) $\sqrt{6}$; (2) $\sqrt{2}$

解析 $\angle B = 45^\circ, \angle C = 60^\circ \Rightarrow \angle A = 75^\circ,$

由正弦定理知, $\frac{\overline{BC}}{\sin A} = \frac{\overline{AB}}{\sin C}$

$$\Rightarrow \overline{AB} = \frac{\overline{BC} \sin C}{\sin A} = \frac{(\sqrt{3}+1) \sin 60^\circ}{\sin 75^\circ} = \frac{(\sqrt{3}+1) \times \frac{\sqrt{3}}{2}}{\frac{\sqrt{6}+\sqrt{2}}{4}} = \sqrt{6},$$

$$\text{又 } 2R = \frac{\overline{AB}}{\sin C} \Rightarrow R = \frac{\overline{AB}}{2 \sin C} = \frac{\sqrt{6}}{2 \cdot \frac{\sqrt{3}}{2}} = \sqrt{2}.$$



9. 設 $\triangle ABC$ 中, $\overline{AB} = 2, \overline{CA} = 1 + \sqrt{3}, \angle A = 30^\circ$, 則:

(1) \overline{BC} 的長度為_____ . (2) $\angle C$ 的大小為_____度 .

解答 (1) $\sqrt{2}$; (2) 45°

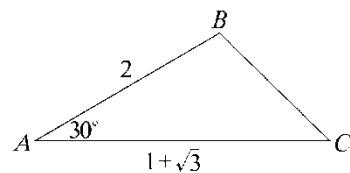
解析 (1) 根據餘弦定理可得

$$\overline{BC}^2 = 2^2 + (1 + \sqrt{3})^2 - 2 \cdot 2 \cdot (1 + \sqrt{3}) \cos 30^\circ = 4 + (4 + 2\sqrt{3}) - 2\sqrt{3} \cdot (1 + \sqrt{3}) = 2,$$

$$\overline{BC} = \sqrt{2}.$$

(2) 因為 $b = 1 + \sqrt{3} > 2 = c$, 故 $\angle C$ 為銳角, 由正弦定理知:

$$\frac{2}{\sin C} = \frac{\sqrt{2}}{\sin 30^\circ} \Rightarrow \sin C = \frac{2}{\sqrt{2}} \cdot \frac{1}{2} = \frac{1}{\sqrt{2}}, \therefore \angle C = 45^\circ.$$



10. $\triangle ABC$ 之三邊長分別為 $\overline{AB} = 5, \overline{BC} = 6, \overline{AC} = 7$, 則:

(1) $\triangle ABC$ 之內切圓半徑為_____ .

(2) 若 $\angle A$ 之外角平分線交直線 BC 於 D , 則 \overline{AD} 長為_____ .

解答 (1) $\frac{2}{3}\sqrt{6}$; (2) $2\sqrt{70}$

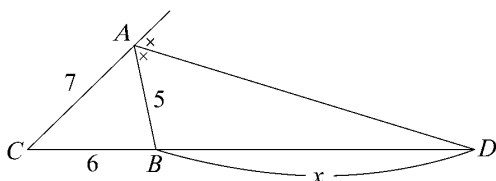
解析 (1) $s = \frac{1}{2}(5 + 6 + 7) = 9, \Delta = \sqrt{s(s-a)(s-b)(s-c)} = \sqrt{9(9-5)(9-6)(9-7)} = 6\sqrt{6},$

$$\text{內切圓半徑 } r = \frac{\Delta}{s} = \frac{6\sqrt{6}}{9} = \frac{2}{3}\sqrt{6}.$$

$$(2) \frac{\overline{CD}}{\overline{BD}} = \frac{\overline{AC}}{\overline{AB}} \Rightarrow \frac{x+6}{x} = \frac{7}{5} \Rightarrow \overline{BD} = x = 15,$$

由 $\triangle ABC$ 及 $\triangle ACD$ 中, 利用餘弦定理可得:

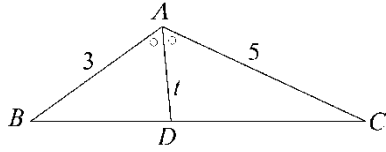
$$\cos C = \frac{7^2 + 6^2 - 5^2}{2 \cdot 7 \cdot 6} = \frac{7^2 + 21^2 - \overline{AD}^2}{2 \cdot 7 \cdot 21} \Rightarrow \overline{AD} = \sqrt{280} = 2\sqrt{70}.$$



11. 設 $\triangle ABC$ 中, $\angle A$ 的分角線交 \overline{BC} 於 D , 已知 $\overline{AB}=3$, $\overline{AC}=5$, $\angle A=120^\circ$, 則 \overline{AD} 的長為_____.

解答 $\frac{15}{8}$

解析



由 $\triangle ABC = \triangle ABD + \triangle ACD$

$$\Rightarrow \frac{1}{2} \cdot 3 \cdot t \cdot \sin 60^\circ + \frac{1}{2} \cdot 5 \cdot t \cdot \sin 60^\circ = \frac{1}{2} \cdot 3 \cdot 5 \cdot \sin 120^\circ \Rightarrow 8t = 15 \Rightarrow t = \frac{15}{8}.$$

12. 設 $\triangle ABC$ 的外接圓半徑為10, 而 $\widehat{AB} : \widehat{BC} : \widehat{CA} = 4:5:3$, 則三角形的面積為_____.

$$(\sin 75^\circ = \frac{\sqrt{6} + \sqrt{2}}{4})$$

解答 $25(3 + \sqrt{3})$

解析 $\widehat{AB} : \widehat{BC} : \widehat{CA} = 4:5:3$

$$\Rightarrow \begin{cases} \angle A = \frac{5}{12} \times 180^\circ = 75^\circ \\ \angle B = \frac{3}{12} \times 180^\circ = 45^\circ \\ \angle C = \frac{4}{12} \times 180^\circ = 60^\circ \end{cases} \Rightarrow \begin{cases} a = 2R \sin A = 20 \cdot \frac{\sqrt{6} + \sqrt{2}}{4} = 5(\sqrt{6} + \sqrt{2}) \\ b = 2R \sin B = 20 \cdot \frac{\sqrt{2}}{2} = 10\sqrt{2} \\ c = 2R \sin C = 20 \cdot \frac{\sqrt{3}}{2} = 10\sqrt{3} \end{cases}$$

$$\Rightarrow \triangle = \frac{1}{2} bcsin A = \frac{1}{2} \cdot 10\sqrt{2} \cdot 10\sqrt{3} \cdot \frac{\sqrt{6} + \sqrt{2}}{4} = 25(3 + \sqrt{3}).$$

13. $\triangle ABC$ 中, $a=2$, $b=3$, $c=4$, h_a 表 a 邊之高, m_c 表 $c = \overline{AB}$ 邊之中線長, 求:

(1) $h_a =$ _____. (2) $m_c =$ _____.

解答 (1) $\frac{3\sqrt{15}}{4}$; (2) $\frac{\sqrt{10}}{2}$

解析 (1) $s = \frac{2+3+4}{2} = \frac{9}{2}$, $\therefore \triangle = \sqrt{s(s-a)(s-b)(s-c)} = \sqrt{\frac{9}{2} \cdot \frac{1}{2} \cdot \frac{3}{2} \cdot \frac{5}{2}} = \frac{3\sqrt{15}}{4}$.

$$(2) \frac{3\sqrt{15}}{4} = \frac{1}{2} \cdot 2 \cdot h_a \Rightarrow h_a = \frac{3\sqrt{15}}{4}.$$

$$(3) m_c = \frac{1}{2} \sqrt{2(a^2 + b^2) - c^2} = \frac{1}{2} \sqrt{2(4+9) - 16} = \frac{\sqrt{10}}{2}.$$

14. 圓內接四邊形 $ABCD$ 中, 若 $\overline{AB}=6$, $\overline{BC}=4$, $\overline{CD}=6$, $\angle B=120^\circ$, 則:

(1) $\overline{AC} =$ _____. (2) $\overline{AD} =$ _____. (3) 四邊形 $ABCD$ 的面積 = _____.

解答 (1)10;(2) $2\sqrt{19}$;(3) $21\sqrt{3}$

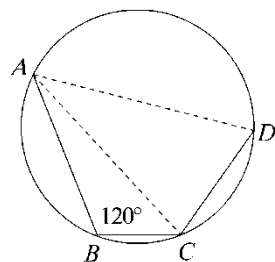
解析 (1)圓內接四邊形 $ABCD$, $\angle B = 120^\circ \Rightarrow \angle D = 60^\circ$, 於 $\triangle ABC$ 中, 利用餘弦定理,

$$\overline{AC}^2 = 6^2 + 4^2 - 2 \cdot 6 \cdot 4 \cdot \cos 120^\circ = 76 \Rightarrow \overline{AC} = \sqrt{76} = 2\sqrt{19} .$$

(2)於 $\triangle ADC$ 中, 設 $\overline{AD} = d$, 再次利用餘弦定理,

$$\overline{AC}^2 = 6^2 + d^2 - 2 \cdot 6 \cdot d \cdot \cos 60^\circ \Rightarrow d^2 - 6d - 40 = 0 \Rightarrow d = 10 .$$

(3)隨之, 四邊形 $ABCD$ 之面積 $= \frac{1}{2}(6 \cdot 4 + 6 \cdot 10) \cdot \frac{\sqrt{3}}{2} = 21\sqrt{3} .$



15.已知 $\triangle ABC$ 內接於半徑為 R 的一個圓, 且 $\overline{AB} = 2$, $\overline{AC} = 3$, $\angle A = 120^\circ$, 則:

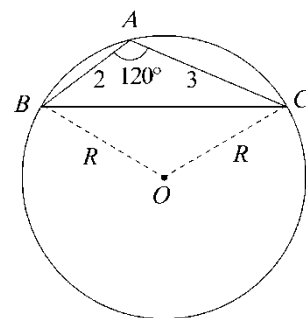
(1) $\overline{BC} =$ _____ . (2) $R =$ _____ .

解答 (1) $\sqrt{19}$;(2) $\frac{\sqrt{57}}{3}$

解析 $\overline{BC}^2 = 2^2 + 3^2 - 2 \cdot 2 \cdot 3 \cdot \cos 120^\circ = 4 + 9 + 6 = 19, \therefore \overline{BC} = \sqrt{19} ,$

在 $\triangle BOC$ 中, $\angle BOC = 120^\circ, \overline{OB} = \overline{OC} = R,$

$$\cos 120^\circ = \frac{R^2 + R^2 - (\sqrt{19})^2}{2 \cdot R \cdot R} \Rightarrow -\frac{1}{2} = \frac{2R^2 - 19}{2R^2} \Rightarrow 3R^2 = 19, \text{ 故 } R = \frac{\sqrt{57}}{3} .$$



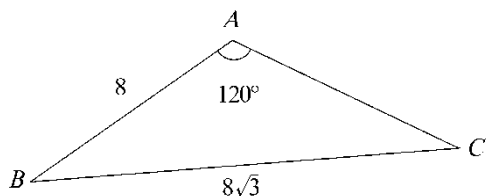
16.在 $\triangle ABC$ 中, $\overline{AB} = 8, \overline{BC} = 8\sqrt{3}, \angle A = 120^\circ$, 則 $\triangle ABC$ 面積 = _____ .

解答 $16\sqrt{3}$

解析 $\frac{\overline{BC}}{\sin A} = \frac{\overline{AB}}{\sin C} \Rightarrow \frac{8\sqrt{3}}{\sqrt{3}} = \frac{8}{\sin C} \Rightarrow \sin C = \frac{1}{2}, \therefore \angle C = 30^\circ \text{ 或 } 150^\circ \text{ (不合)},$

$$\angle B = 180^\circ - \angle A - \angle C = 30^\circ ,$$

$$\therefore \triangle ABC \text{ 面積} = \frac{1}{2} \overline{AB} \cdot \overline{BC} \cdot \sin B = \frac{1}{2} \cdot 8 \cdot 8\sqrt{3} \cdot \sin 30^\circ = 16\sqrt{3} .$$



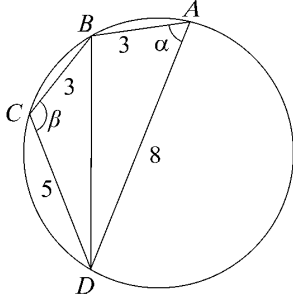
17.已知圓內接四邊形 $ABCD$, $\overline{AB} = \overline{BC} = 3, \overline{CD} = 5, \overline{DA} = 8$, 則 $\overline{BD} =$ _____ .

解答 7

解析 利用餘弦定理, 則 $\begin{cases} \overline{BD}^2 = 8^2 + 3^2 - 2 \times 8 \times 3 \times \cos \alpha \cdots \cdots \textcircled{1} \\ \overline{BD}^2 = 5^2 + 3^2 - 2 \times 5 \times 3 \times \cos \beta \cdots \cdots \textcircled{2} \end{cases}$,

$$\begin{aligned} \textcircled{1} = \textcircled{2} &\Rightarrow 64 - 48\cos\alpha = 25 - 30\cos\beta, \\ 64 - 48\cos\alpha &= 25 - 30(-\cos\alpha) \quad (\because \alpha + \beta = 180^\circ), \\ 64 - 48\cos\alpha &= 25 + 30\cos\alpha \Rightarrow \cos\alpha = \frac{1}{2} \text{ 代入 } \textcircled{1}, \end{aligned}$$

$$\overline{BD}^2 = 8^2 + 3^2 - 2 \times 8 \times 3 \times \frac{1}{2} = 49, \therefore \overline{BD} = 7.$$



18. $\triangle ABC$ 中, 若 $(a+b+c)(b+c-a) = 3bc$, 則: (1) $\cos A =$ _____ . (2) $\angle A =$ _____ .

解答 (1) $\frac{1}{2}$; (2) 60°

解析 $(a+b+c)(b+c-a) = 3bc \Rightarrow (b+c)^2 - a^2 = 3bc \Rightarrow b^2 + c^2 - a^2 = bc,$
 $\cos A = \frac{b^2 + c^2 - a^2}{2bc} = \frac{bc}{2bc} = \frac{1}{2} \Rightarrow \angle A = 60^\circ.$

19. 四邊形的兩對角線長為 8 與 6, 若兩對角線夾角為 α, β , 且 $\alpha = 5\beta$, 則此四邊形之面積為 _____ .

解答 12

解析 $\alpha + \beta = 180^\circ \Rightarrow 6\beta = 180^\circ \Rightarrow \beta = 30^\circ$, 四邊形面積 $= \frac{1}{2} \times 8 \times 6 \times \sin 30^\circ = 12.$

20. $\triangle ABC$ 之周長為 20, 內切圓半徑為 $\sqrt{3}$, $\angle BAC = 60^\circ$, 則 $\triangle ABC$ 之外接圓面積為 _____ .

解答 $\frac{49}{3}\pi$

解析 $\triangle = rs = \sqrt{3} \times \frac{20}{2} = 10\sqrt{3} \Rightarrow \triangle = \frac{1}{2}bc \sin 60^\circ = 10\sqrt{3}, \therefore bc = 40,$

$$\text{又 } a^2 = b^2 + c^2 - 2bc \cdot \cos 60^\circ \Rightarrow \begin{cases} a^2 = b^2 + c^2 - 40 \dots \textcircled{1} \\ a + b + c = 20 \dots \textcircled{2} \end{cases},$$

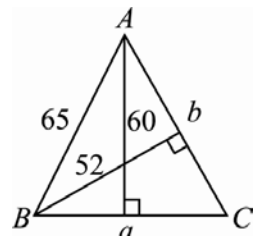
$$\text{由 } \textcircled{2} \Rightarrow (b+c)^2 = (20-a)^2 \Rightarrow b^2 + c^2 + 2bc = a^2 - 40a + 400,$$

$$\text{由 } \textcircled{1} \Rightarrow a^2 + 40 + 2 \times 40 = a^2 - 40a + 400, \therefore a = 7$$

$$\Rightarrow R = \frac{abc}{4\triangle} = \frac{7(40)}{4(10\sqrt{3})} = \frac{7}{\sqrt{3}}, \therefore \triangle ABC \text{ 的外接圓面積} = \left(\frac{7}{\sqrt{3}}\right)^2 \pi = \frac{49}{3}\pi.$$

21. 三角形 ABC 中, $h_a = 60, h_b = 52, c = 65$, 求 $h_c =$ _____ .

(h_a, h_b, h_c , 分別表過 A, B, C 的高)



解答 $\frac{390}{7}$

解析 $a:b = \frac{1}{60} : \frac{1}{52} = 13:15$, 設 $a = 13t$, $b = 15t$,

$$\therefore \Delta = \frac{1}{2} \times 65 \times h_c = \frac{1}{2} \times 13t \times 60, \therefore h_c = 12t,$$

$$\text{又 } \Delta = \frac{1}{2} (65)(12t) = \sqrt{\left(\frac{65}{2} + 14t\right)\left(\frac{65}{2} + t\right)\left(\frac{65}{2} - t\right)\left(14t - \frac{65}{2}\right)}$$

$$\Rightarrow 16 \times 196t^4 - 4 \times 53 \times 65^2 \times t^2 + 65^4 = 0$$

$$\Rightarrow t = \frac{65}{4} \text{ (不合) 或 } \frac{65}{14}, \therefore h_c = 12t = 12\left(\frac{65}{14}\right) = \frac{390}{7}.$$

22. $\triangle ABC$ 之周長為 20, $\angle B = 60^\circ$, 外接圓半徑為 $\frac{7\sqrt{3}}{3}$, 則內切圓半徑為_____.

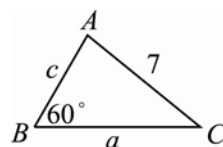
解答 $\sqrt{3}$

解析 由正弦定理: $\frac{b}{\sin 60^\circ} = 2\left(\frac{7\sqrt{3}}{3}\right) \Rightarrow b = 7$,

$$\text{由餘弦定理: } 49 = a^2 + c^2 - 2ac \cdot \cos 60^\circ, 49 = (a+c)^2 - 2ac - ac,$$

$$\therefore a+b+c = a+7+c = 20, \therefore a+c = 13, \therefore 49 = (13)^2 - 3ac \Rightarrow ac = 40,$$

$$\triangle ABC \text{ 面積} = \frac{1}{2}(40)\left(\frac{\sqrt{3}}{2}\right) = 10\sqrt{3}, \text{ 則內切圓半徑 } r = \frac{\triangle ABC}{s} = \frac{10\sqrt{3}}{10} = \sqrt{3}.$$



23. 如圖, $\angle ABD = 90^\circ$, $\overline{AB} = 5$, $\overline{BD} = 10$, $\overline{AC} = 4$, $\overline{BC} = 3$, 則:

(1) $\overline{CD} =$ _____. (2) $\triangle ACD$ 面積 = _____.

解答 (1) $\sqrt{61}$; (2) 10

解析 由條件 $\Rightarrow \angle ACB = 90^\circ$, 又 $\angle ABD = 90^\circ$,

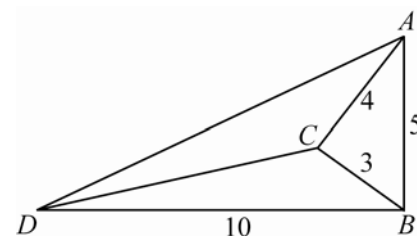
$$\therefore \angle BAC = \angle CBD = \theta, \therefore \cos \theta = \frac{4}{5}, \sin \theta = \frac{3}{5}.$$

$$(1) \text{ 在 } \triangle CBD \text{ 中, } \overline{CD}^2 = 10^2 + 3^2 - 2 \times 10 \times 3 \times \cos \theta = 100 + 9 - 2 \times 10 \times 3 \times \frac{4}{5} = 61,$$

$$\therefore \overline{CD} = \sqrt{61}.$$

$$(2) \triangle ACD \text{ 面積} = \triangle ABD \text{ 面積} - \triangle ABC \text{ 面積} - \triangle BCD \text{ 面積}$$

$$= \frac{1}{2} \times 10 \times 5 - \frac{1}{2} \times 4 \times 3 - \frac{1}{2} \times 10 \times 3 \times \sin \theta = 25 - 6 - 15 \times \frac{3}{5} = 10.$$



24. (1) Δ 三高長 3、4、5, 求 Δ 面積為_____。

(2) Δ 三中線長 3、4、5, 求 Δ 面積為_____。

解答 (1) $\frac{120\sqrt{128639}}{128639}$; (2) 8

解析 (1) $h_a = 5, h_b = 4, h_c = 3 \Rightarrow a:b:c = \frac{1}{h_a} : \frac{1}{h_b} : \frac{1}{h_c} = \frac{1}{5} : \frac{1}{4} : \frac{1}{3} = 12:15:20$

$$\text{設 } a = 12t, b = 15t, c = 20t \Rightarrow \cos A = \frac{15^2 + 20^2 - 12^2}{2 \times 15 \times 20} = \frac{481}{600}, \sin A = \frac{\sqrt{128639}}{600}$$

$$\text{又 } \sin A = \frac{h_c}{b} = \frac{3}{15t} = \frac{1}{5t}, \text{ 即}$$

$$\frac{\sqrt{128639}}{600} = \frac{1}{5t} \Rightarrow t = \frac{120}{\sqrt{128639}} = \frac{120\sqrt{128639}}{128639} = \frac{120\sqrt{128639}}{128639}$$

$$a = 12t = \frac{480\sqrt{3129}}{1043} \Rightarrow \Delta = \frac{1}{2} \times \frac{480\sqrt{3129}}{1043} \times 5 = \frac{1200\sqrt{3129}}{1043}$$

$$(2) \Delta = \frac{4}{3} \Delta_m = \frac{4}{3} \sqrt{6 \cdot 3 \cdot 2 \cdot 1} = 8$$