

高雄市明誠中學 高二數學平時測驗 日期：100.09.06				
範圍	1-1 銳角三角函數	班級	二年__班	姓名
		座號		

一、填充題 (每題 10 分)

1. 設  $\theta$  為銳角, 且  $\tan \theta = \frac{\sqrt{5}}{2}$ , 則  $\frac{\sin \theta}{1 + \cos \theta} =$  \_\_\_\_\_ .

**解答**  $\frac{\sqrt{5}}{5}$

**解析** 因為  $\theta$  為銳角, 且  $\tan \theta = \frac{\sqrt{5}}{2} \Rightarrow r = \sqrt{2^2 + (\sqrt{5})^2} = 3$ , 所以  $\cos \theta = \frac{2}{3}$ ,  $\sin \theta = \frac{\sqrt{5}}{3}$ ,

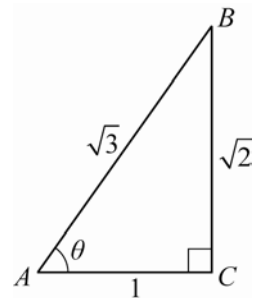
$$\text{故 } \frac{\sin \theta}{1 + \cos \theta} = \frac{\frac{\sqrt{5}}{3}}{1 + \frac{2}{3}} = \frac{\sqrt{5}}{5} .$$

2. 設  $\theta$  為銳角且  $\tan \theta = \sqrt{2}$ , 則: (1)  $\sin \theta =$  \_\_\_\_\_ . (2)  $\cos \theta =$  \_\_\_\_\_ .

**解答** (1)  $\frac{\sqrt{6}}{3}$ ; (2)  $\frac{\sqrt{3}}{3}$

**解析** 如圖:  $\tan \theta = \frac{\sqrt{2}}{1} \Rightarrow r = \sqrt{1^2 + (\sqrt{2})^2} = \sqrt{3}$ ,

$$\sin A = \frac{\overline{BC}}{\overline{AB}} = \frac{\sqrt{2}}{\sqrt{3}} = \frac{\sqrt{6}}{3}, \quad \cos A = \frac{\overline{AC}}{\overline{AB}} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3} .$$



3. 設四邊形  $ABCD$  中,  $\overline{AB} = 3$ ,  $\overline{BC} = 2$ ,  $\overline{CD} = 1$ , 且  $\angle ABD = \angle BCD = 90^\circ$ , 則:

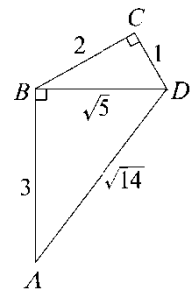
(1)  $\sin A =$  \_\_\_\_\_ . (2)  $\cos A =$  \_\_\_\_\_ .

**解答** (1)  $\sin A = \frac{\sqrt{5}}{\sqrt{14}}$ ; (2)  $\cos A = \frac{3}{\sqrt{14}}$

**解析**

如圖: 四邊形  $ABCD$  中,  $\overline{AB} = 3$ ,  $\overline{BC} = 2$ ,  $\overline{CD} = 1$ , 且  $\angle ABD = \angle BCD = 90^\circ$ , 故得  $\overline{BD} = \sqrt{5}$ ,  $\overline{AD} = \sqrt{14}$ ,

由定義  $\sin A = \frac{\sqrt{5}}{\sqrt{14}}$ ,  $\cos A = \frac{3}{\sqrt{14}}$  .



4.  $\frac{2 \sin 60^\circ \cos 30^\circ - \sin^2 45^\circ \tan^2 60^\circ + \tan 45^\circ}{\sin 30^\circ \cos 60^\circ - \cos^2 45^\circ \tan^2 30^\circ}$  之值為 \_\_\_\_\_ .

**解答** 12

**解析** 原式 =  $\frac{2 \cdot \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{3}}{2} - (\frac{\sqrt{2}}{2})^2 \cdot (\sqrt{3})^2 + 1}{\frac{1}{2} \cdot \frac{1}{2} - (\frac{\sqrt{2}}{2})^2 \cdot (\frac{\sqrt{3}}{3})^2} = \frac{1}{\frac{1}{4} - \frac{1}{6}} = 12 .$

5. 設函數  $f(x) = 4 \sin x$ ,  $30^\circ \leq x \leq 60^\circ$ , 則  $f(x)$  的範圍為 \_\_\_\_\_ .

**解答**  $2 \leq f(x) \leq 2\sqrt{3}$

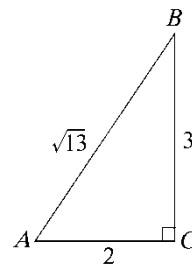
**解析** 正弦函數在  $0^\circ < x < 90^\circ$  是遞增函數，故當  $30^\circ \leq x \leq 60^\circ$  時，我們可得

$$\sin 30^\circ \leq \sin x \leq \sin 60^\circ \Rightarrow 4 \cdot \frac{1}{2} \leq 4 \sin x \leq 4 \cdot \frac{\sqrt{3}}{2}, \text{ 所以 } 2 \leq f(x) \leq 2\sqrt{3} .$$

6.  $\triangle ABC$  中， $\angle C = 90^\circ$ ， $\overline{AC} = 2$ ， $\overline{AB} = \sqrt{13}$ ，則：(1)  $\sin B =$  \_\_\_\_\_ . (2)  $\cos B =$  \_\_\_\_\_ .

**解答** (1)  $\frac{2}{\sqrt{13}}$ ; (2)  $\frac{3}{\sqrt{13}}$

**解析**  $\overline{BC} = \sqrt{(\sqrt{13})^2 - 2^2} = 3$ ， $\therefore \sin B = \frac{2}{\sqrt{13}}$ ， $\cos B = \frac{3}{\sqrt{13}}$  .



7.  $\sin 30^\circ \cdot \cos 45^\circ \cdot \tan 60^\circ \cdot \cot 45^\circ =$  \_\_\_\_\_ .

**解答**  $\frac{\sqrt{6}}{4}$

**解析** 原式  $= \frac{1}{2} \cdot \frac{1}{\sqrt{2}} \cdot \sqrt{3} \cdot 1 = \frac{\sqrt{6}}{4}$  .

8.  $\cos^2 40^\circ + \cos^2 50^\circ + \cos^2 60^\circ =$  \_\_\_\_\_ .

**解答**  $\frac{5}{4}$

**解析** 原式  $= \cos^2 40^\circ + \cos^2(90^\circ - 40^\circ) + \cos^2 60^\circ = \cos^2 40^\circ + \sin^2 40^\circ + (\frac{1}{2})^2 = 1 + \frac{1}{4} = \frac{5}{4}$  .

9.  $\sin^2(45^\circ + \theta) + \sin^2(45^\circ - \theta) =$  \_\_\_\_\_ .

**解答** 1

**解析** 原式  $= \sin^2(45^\circ + \theta) + \sin^2(90^\circ - (45^\circ + \theta)) = \sin^2(45^\circ + \theta) + \cos^2(45^\circ + \theta) = 1$  .

10.  $\log_2 \sin 30^\circ + \log_2 \cos 30^\circ + \log_2 \tan 30^\circ =$  \_\_\_\_\_ .

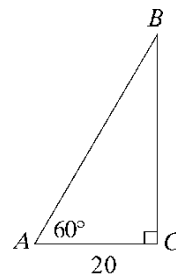
**解答** -2

**解析** 原式  $= \log_2(\sin 30^\circ \cos 30^\circ \tan 30^\circ) = \log_2(\frac{1}{2} \cdot \frac{\sqrt{3}}{2} \cdot \frac{1}{\sqrt{3}}) = \log_2(\frac{1}{2})^2 = -2$  .

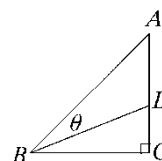
11. 在直角  $\triangle ABC$  中， $\angle C = 90^\circ$ ， $\angle A = 60^\circ$ ， $\overline{AC} = 20$ ，求  $\overline{BC} =$  \_\_\_\_\_ .

**解答**  $20\sqrt{3}$

**解析** 如圖， $\angle A = 60^\circ$ ，則  $\frac{\sqrt{3}}{1} = \frac{\overline{BC}}{20} \Rightarrow \overline{BC} = 20\sqrt{3}$  .

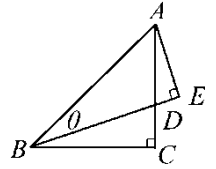


12. 如圖： $\overline{AC} = \overline{BC}$ ， $\overline{AD} : \overline{DC} = 3 : 2$ ，則  $\tan \theta =$  \_\_\_\_\_ .



**解答**  $\frac{3}{7}$

**解析** 設  $\overline{AD} = 3$ ,  $\overline{DC} = 2$ ,  $\overline{BC} = 5$ , 作  $\overline{AE}$  垂直  $\overline{BD}$  延長線於  $E$ ,  
因  $\triangle AED \sim \triangle BCD$ , 故  $\overline{AE} : \overline{DE} = \overline{BC} : \overline{CD} = 5 : 2$ , 可設  $\overline{AE} = 5t$ ,  $\overline{DE} = 2t$ ,



$$\overline{AD} = \sqrt{(5t)^2 + (2t)^2} = \sqrt{29}t = 3 \Rightarrow t = \frac{3}{\sqrt{29}} \Rightarrow \tan \theta = \frac{\overline{AE}}{\overline{BE}} = \frac{\frac{15}{\sqrt{29}}}{\sqrt{29} + \frac{6}{\sqrt{29}}} = \frac{15}{35} = \frac{3}{7}.$$

13.  $\triangle ABC$  中,  $\overline{AB} = 5$ ,  $\overline{BC} = 3$ ,  $\overline{CA} = 4$ ,  $\angle B$  的分角線交  $\overline{AC}$  於  $D$ , 則:

(1)  $\cos B =$  \_\_\_\_\_ . (2)  $\tan \angle DBC =$  \_\_\_\_\_ .

**解答** (1)  $\frac{3}{5}$ ; (2)  $\frac{1}{2}$

**解析** (1)  $\triangle ABC$  中,  $\overline{BC}^2 + \overline{CA}^2 = 9 + 16 = 25 = \overline{AB}^2$ ,  $\therefore \angle C = 90^\circ \Rightarrow \cos B = \frac{\overline{BC}}{\overline{AB}} = \frac{3}{5}$ .

(2)  $\because \overline{BD}$  為  $\angle B$  之分角線,  $\therefore \overline{AD} : \overline{DC} = \overline{AB} : \overline{BC} = 5 : 3 \Rightarrow \overline{CD} = 4 \times \frac{3}{8} = \frac{3}{2}$ ,

$$\text{故 } \tan \angle DBC = \frac{\overline{CD}}{\overline{BC}} = \frac{\frac{3}{2}}{3} = \frac{1}{2}.$$

14. 設  $0^\circ < \theta < 90^\circ$  且  $3\sin \theta + \cos \theta = 2$ , 則  $\sin \theta + \cos \theta =$  \_\_\_\_\_ .

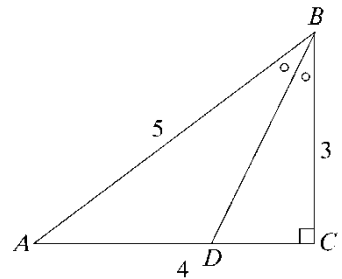
**解答**  $\frac{4 + \sqrt{6}}{5}$

**解析**  $3\sin \theta + \cos \theta = 2 \Rightarrow \cos \theta = 2 - 3\sin \theta$  代入  $\sin^2 \theta + \cos^2 \theta = 1$

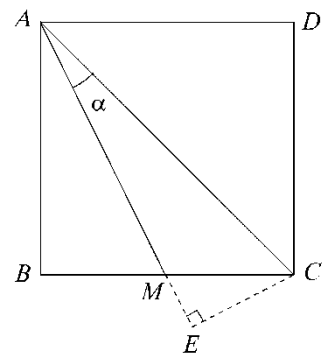
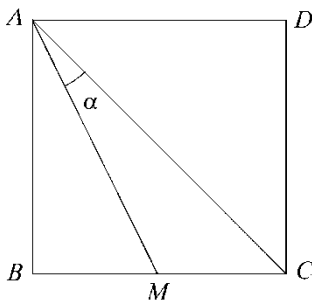
$$\Rightarrow 10\sin^2 \theta - 12\sin \theta + 3 = 0 \Rightarrow \sin \theta = \frac{6 \pm \sqrt{6}}{10},$$

$$\therefore \cos \theta = 2 - 3\sin \theta = \frac{2 \mp 3\sqrt{6}}{10} \quad (\text{其中 } \frac{2 - 3\sqrt{6}}{10} < 0 \text{ 不合}),$$

$$\therefore \sin \theta + \cos \theta = \frac{6 - \sqrt{6}}{10} + \frac{2 + 3\sqrt{6}}{10} = \frac{4 + \sqrt{6}}{5}.$$



15. 如圖, 正方形  $ABCD$  中,  $M$  為  $\overline{BC}$  中點,  $\angle MAC = \alpha$ , 則  $\tan \alpha =$  \_\_\_\_\_ .



**解答**  $\frac{1}{3}$

**解析** 過  $C$  作  $\overline{CE}$  垂直  $\overline{AM}$  的延長線於點  $E$ , 設正方形邊長為  $a$ , 由相似性質知  $\triangle ABM \sim \triangle CEM$ ,

$$\therefore \frac{\overline{CE}}{\overline{AB}} = \frac{\overline{EM}}{\overline{BM}} = \frac{\overline{CM}}{\overline{AM}}, \text{ 故 } \frac{\overline{CE}}{a} = \frac{\overline{EM}}{\frac{1}{2}a} = \frac{\frac{1}{2}a}{\frac{\sqrt{5}}{2}a}, \text{ 即 } \overline{CE} = \frac{a}{\sqrt{5}}, \overline{EM} = \frac{a}{2\sqrt{5}},$$

$$\text{在 } \triangle AEC \text{ 中, } \because \angle AEC = 90^\circ, \text{ 故 } \tan \alpha = \frac{\overline{CE}}{\overline{AE}} = \frac{\frac{a}{\sqrt{5}}}{\frac{\sqrt{5}a}{2} + \frac{a}{2\sqrt{5}}} = \frac{1}{3}.$$

16. 直角  $\triangle ABC$  中,  $\overline{AB} = c$ ,  $\overline{BC} = a$ ,  $\overline{CA} = b$ ,  $\angle C = 90^\circ$ , 若  $\frac{3}{5} \cos A + \cos B = 1$ , 則

$a : b : c =$  \_\_\_\_\_ .

**解答**  $8 : 15 : 17$

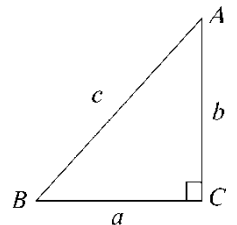
**解析** 原式  $\Rightarrow \frac{3}{5} \cdot \frac{b}{c} + \frac{a}{c} = 1 \Rightarrow 3b + 5a = 5c \Rightarrow 3b = 5c - 5a$ ,

$$\text{又 } a^2 + b^2 = c^2 \Rightarrow 9a^2 + 25(c-a)^2 = 9c^2 \Rightarrow 34a^2 - 50ca + 16c^2 = 0$$

$$\Rightarrow 17a^2 - 25ca + 8c^2 = 0$$

$$\Rightarrow (17a - 8c)(a - c) = 0 \Rightarrow a = \frac{8}{17}c \text{ 或 } c \text{ (不合)},$$

$$\therefore 3b = \frac{-40c}{17} + 5c = \frac{45c}{17} \Rightarrow b = \frac{15}{17}c, \therefore a : b : c = \frac{8c}{17} : \frac{15c}{17} : c = 8 : 15 : 17.$$



17. 設等腰  $\triangle ABC$  中,  $\angle B = 90^\circ$ , 若  $D$  是  $\overline{BC}$  的中點, 則:

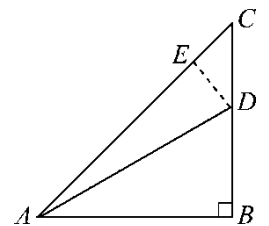
(1)  $\tan \angle BAD =$  \_\_\_\_\_ . (2)  $\tan \angle CAD =$  \_\_\_\_\_ .

**解答** (1)  $\frac{1}{2}$ ; (2)  $\frac{1}{3}$

**解析** 如圖: 在  $\triangle ABC$  中, 設  $\overline{AB} = \overline{BC} = 2\sqrt{2}$ , 又  $\angle B = 90^\circ$ ,  $\overline{AC} = \sqrt{2} \overline{BC} = 4$ , 過  $D$  作  $\overline{DE} \perp \overline{AC}$ , 垂足為  $E$ , 已知  $D$  為  $\overline{BC}$  中點,

$$\text{故知 } \overline{DE} = \overline{EC} = \frac{\sqrt{2}}{2} \overline{CD} = \frac{\sqrt{2}}{2} \cdot \sqrt{2} = 1, \overline{AE} = \overline{AC} - \overline{EC} = 4 - 1 = 3,$$

$$\tan \angle BAD = \frac{\overline{BD}}{\overline{AB}} = \frac{\sqrt{2}}{2\sqrt{2}} = \frac{1}{2}, \tan \angle CAD = \frac{\overline{DE}}{\overline{AE}} = \frac{1}{3}.$$



18.  $4\cos^2 30^\circ + 2\sin^2 45^\circ + \tan^2 45^\circ + \tan^2 60^\circ + 4\cos^2 60^\circ + 4\sin^2 60^\circ =$  \_\_\_\_\_ .

**解答** 12

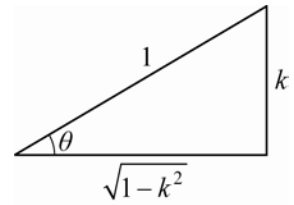
**解析** 原式  $= 4\left(\frac{\sqrt{3}}{2}\right)^2 + 2\left(\frac{\sqrt{2}}{2}\right)^2 + 1^2 + (\sqrt{3})^2 + 4\left(\frac{1}{2}\right)^2 + 4\left(\frac{\sqrt{3}}{2}\right)^2 = 3 + 1 + 1 + 3 + 1 + 3 = 12.$

19. 設  $\theta$  是一個銳角， $\sin\theta = k$ ，以  $k$  表出：

(1)  $\tan\theta =$  \_\_\_\_\_ . (2)  $\cos\theta =$  \_\_\_\_\_ .

**解答** (1)  $\frac{k}{\sqrt{1-k^2}}$ ; (2)  $\sqrt{1-k^2}$

**解析**  $\sin\theta = k = \frac{k}{1}$ ，作圖，則  $\tan\theta = \frac{k}{\sqrt{1-k^2}}$ ， $\cos\theta = \sqrt{1-k^2}$  .



20. 設  $\sin\theta + \cos\theta = \frac{5}{4}$ ，求  $\sin\theta \cdot \cos\theta =$  \_\_\_\_\_ .

**解答**  $\frac{9}{32}$

**解析**  $(\sin\theta + \cos\theta)^2 = (\frac{5}{4})^2 \Rightarrow (\sin^2\theta + \cos^2\theta) + 2\sin\theta\cos\theta = \frac{25}{16}$   
 $\Rightarrow 1 + 2\sin\theta\cos\theta = \frac{25}{16} \Rightarrow \sin\theta\cos\theta = \frac{9}{32}$  .

21. 設  $\theta$  為銳角，若  $\sin\theta\cos\theta = \frac{1}{4}$ ，則  $\sin\theta - \cos\theta$  之值為 \_\_\_\_\_ .

**解答**  $\pm \frac{\sqrt{2}}{2}$

**解析**  $\because (\sin\theta - \cos\theta)^2 = \sin^2\theta - 2\sin\theta\cos\theta + \cos^2\theta = 1 - 2\sin\theta\cos\theta = 1 - 2 \times \frac{1}{4} = \frac{1}{2}$ ，  
 $\therefore \sin\theta - \cos\theta = \pm \frac{1}{\sqrt{2}} = \pm \frac{\sqrt{2}}{2}$  .

22. 設  $\theta$  為銳角， $\sin\theta - \cos\theta = \frac{1}{5}$ ，求  $\sin\theta =$  \_\_\_\_\_ .

**解答**  $\frac{4}{5}$

**解析**  $\sin\theta - \cos\theta = \frac{1}{5} \Rightarrow (\sin\theta - \cos\theta)^2 = (\frac{1}{5})^2 \Rightarrow \sin^2\theta - 2\sin\theta\cos\theta + \cos^2\theta = \frac{1}{25}$ ，  
 $\Rightarrow 1 - 2\sin\theta\cos\theta = \frac{1}{25} \Rightarrow \sin\theta\cos\theta = \frac{12}{25}$ ，

$(\sin\theta + \cos\theta)^2 = \sin^2\theta + 2\sin\theta\cos\theta + \cos^2\theta = 1 + 2 \times \frac{12}{25} = \frac{49}{25}$ ，

又  $\theta$  為一銳角，故  $\sin\theta + \cos\theta = \frac{7}{5}$ ，  

$$\begin{cases} \sin\theta - \cos\theta = \frac{1}{5} \cdots \cdots \textcircled{1} \\ \sin\theta + \cos\theta = \frac{7}{5} \cdots \cdots \textcircled{2} \end{cases}$$

由  $\textcircled{1} + \textcircled{2}$  得  $2\sin\theta = \frac{8}{5}$ ，故  $\sin\theta = \frac{4}{5}$  .

23. 設  $0^\circ < x < 45^\circ$  且  $\sin x + \cos x = \frac{4}{3}$ , 則 :

(1)  $\sin x \cdot \cos x =$  \_\_\_\_\_ . (2)  $\sin x - \cos x =$  \_\_\_\_\_ . (3)  $\tan \theta + \cot \theta =$  \_\_\_\_\_ .

**解答** (1)  $\frac{7}{18}$ ; (2)  $-\frac{\sqrt{2}}{3}$ ; (3)  $\frac{18}{7}$

**解析** (1)  $\sin x + \cos x = \frac{4}{3}$  平方之, 得  $1 + 2\sin x \cos x = \frac{16}{9}$ ,  $\therefore 2\sin x \cos x = \frac{7}{9}$ , 即  $\sin x \cos x = \frac{7}{18}$ .

(2) 因爲  $0^\circ < x < 45^\circ$ , 故  $0 < \sin x < \cos x$ ,

又  $(\sin x - \cos x)^2 = 1 - 2\sin x \cos x = 1 - \frac{7}{9} = \frac{2}{9}$ , 得  $\sin x - \cos x = -\frac{\sqrt{2}}{3}$ .

(3)  $\tan \theta + \cot \theta = \frac{1}{\sin \theta \cos \theta} = \frac{18}{7}$

24. 求  $\cos^2 10^\circ + \cos^2 20^\circ + \cos^2 30^\circ + \cos^2 40^\circ + \cos^2 50^\circ + \cos^2 60^\circ + \cos^2 70^\circ + \cos^2 80^\circ$  的值 = \_\_\_\_\_ .

**解答** 4

**解析** 原式  $= (\cos^2 10^\circ + \cos^2 80^\circ) + (\cos^2 20^\circ + \cos^2 70^\circ) + (\cos^2 30^\circ + \cos^2 60^\circ) + (\cos^2 40^\circ + \cos^2 50^\circ)$   
 $= (\sin^2 80^\circ + \cos^2 80^\circ) + (\sin^2 70^\circ + \cos^2 70^\circ) + (\sin^2 60^\circ + \cos^2 60^\circ) + (\sin^2 50^\circ + \cos^2 50^\circ)$   
 $= 1 + 1 + 1 + 1 = 4$ .

25. 設  $\theta, \phi$  爲銳角, 若  $\theta + \phi = 90^\circ$ , 則  $\sin \theta \cos \phi + \cos \theta \sin \phi =$  \_\_\_\_\_ .

**解答** 1

**解析**  $\because \theta + \phi = 90^\circ, \therefore \phi = 90^\circ - \theta$ , 故  $\sin \theta \cos \phi + \cos \theta \sin \phi$   
 $= \sin \theta \cos(90^\circ - \theta) + \cos \theta \sin(90^\circ - \theta)$   
 $= \sin \theta \sin \theta + \cos \theta \cos \theta$   
 $= \sin^2 \theta + \cos^2 \theta = 1$ .

26.  $\log_8 \sqrt{2 + \tan 60^\circ} + \log_8 \sqrt{1 - \cos 30^\circ} =$  \_\_\_\_\_ .

**解答**  $-\frac{1}{6}$

**解析** 原式  $= \log_8 \sqrt{2 + \sqrt{3}} + \log_8 \sqrt{1 - \frac{\sqrt{3}}{2}} = \log_8 \sqrt{2 + \sqrt{3}} + \log_8 \sqrt{\frac{2 - \sqrt{3}}{2}}$   
 $= \log_8 \sqrt{(2 + \sqrt{3}) \left( \frac{2 - \sqrt{3}}{2} \right)} = \log_8 \sqrt{\frac{1}{2}} = \log_{2^3} 2^{-\frac{1}{2}} = \frac{-1}{3} \log_2 2 = -\frac{1}{6}$ .

27. 設  $f(n) = \sin^n \theta + \cos^n \theta, n \in \mathbf{N}$ , 則  $2f(6) - 3f(4) + 2f(2) =$  \_\_\_\_\_ .

**解答** 1

**解析** 參考公式:  $a^2 + b^2 = (a+b)^2 - 2ab$ ;  $a^3 + b^3 = (a+b)^3 - 3ab(a+b)$   
 $f(6) = \sin^6 \theta + \cos^6 \theta = (\sin^2 \theta + \cos^2 \theta)^3 - 3\sin^2 \theta \cos^2 \theta (\sin^2 \theta + \cos^2 \theta) = 1 - 3\sin^2 \theta \cos^2 \theta$ ,  
 $f(4) = \sin^4 \theta + \cos^4 \theta = (\sin^2 \theta + \cos^2 \theta)^2 - 2\sin^2 \theta \cos^2 \theta = 1 - 2\sin^2 \theta \cos^2 \theta$ ,  
 $f(2) = \sin^2 \theta + \cos^2 \theta = 1$ ,  
 $\therefore 2f(6) - 3f(4) + 2f(2) = 2 - 6\sin^2 \theta \cos^2 \theta - 3 + 6\sin^2 \theta \cos^2 \theta + 2 = 1$ .

28.  $\theta$  為銳角，若  $\sin\theta - \cos\theta = \frac{1}{2}$ ，則：

(1)  $\sin^4\theta + \cos^4\theta =$  \_\_\_\_\_ . (2)  $\tan^2\theta + \frac{1}{\tan^2\theta} =$  \_\_\_\_\_ .

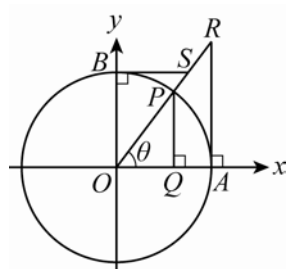
**解答** (1)  $\frac{23}{32}$ ; (2)  $\frac{46}{9}$

**解析**  $\sin\theta - \cos\theta = \frac{1}{2}$  平方之  $\Rightarrow 1 - 2\sin\theta\cos\theta = \frac{1}{4} \Rightarrow \sin\theta\cos\theta = \frac{3}{8}$  .

(1)  $\sin^4\theta + \cos^4\theta = (\sin^2\theta + \cos^2\theta)^2 - 2\sin^2\theta\cos^2\theta = 1 - 2 \cdot \frac{9}{64} = \frac{23}{32}$  .

(2)  $\tan^2\theta + \frac{1}{\tan^2\theta} = \frac{\sin^2\theta}{\cos^2\theta} + \frac{\cos^2\theta}{\sin^2\theta} = \frac{\sin^4\theta + \cos^4\theta}{\sin^2\theta\cos^2\theta} = \frac{\frac{23}{32}}{\frac{9}{64}} = \frac{46}{9}$  .

29. 設  $A(1, 0)$ ， $B(0, 1)$  及  $P$  皆在單位圓上（如圖），且  $\angle POA$  為銳角，過  $P$  作  $\overline{OA}$  的垂線，而交  $\overline{OA}$  於  $Q$ ，延長  $\overline{OP}$  而交過  $A$  的切線於  $R$ ，交過  $B$  的切線於  $S$ ，若  $\overline{BS} = \frac{3}{4}$ ，則  $\triangle OPQ$  的周長 = \_\_\_\_\_ .



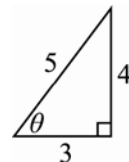
**解答**  $\frac{12}{5}$

**解析**  $\angle OSB = \theta \Rightarrow \tan\theta = \frac{\overline{OB}}{\overline{BS}}$ ，即  $\tan\theta = \frac{1}{\frac{3}{4}} = \frac{4}{3}$ ，

$\cos\theta = \frac{\overline{OQ}}{\overline{OP}} \Rightarrow \overline{OQ} = \overline{OP} \cdot \cos\theta = 1 \cdot \frac{3}{5} = \frac{3}{5}$ ，

$\sin\theta = \frac{\overline{PQ}}{\overline{OP}} \Rightarrow \overline{PQ} = \overline{OP} \cdot \sin\theta = 1 \cdot \frac{4}{5} = \frac{4}{5}$ ，

$\triangle OPQ$  的周長 =  $\frac{3}{5} + \frac{4}{5} + 1 = \frac{12}{5}$  .



30. 若  $\tan\theta = \cos\theta$ ， $\theta$  為銳角，求  $\frac{1}{1-\sin\theta} + \frac{1}{1+\sin\theta} =$  \_\_\_\_\_ .

**解答**  $1 + \sqrt{5}$

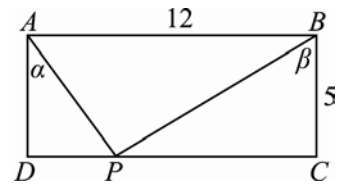
**解析**  $\tan\theta = \cos\theta$ ， $\therefore \frac{\sin\theta}{\cos\theta} = \cos\theta$ ， $\therefore \sin\theta = \cos^2\theta = 1 - \sin^2\theta \Rightarrow \sin^2\theta + \sin\theta - 1 = 0$ ，

$\therefore \sin\theta = \frac{-1 \pm \sqrt{5}}{2}$ （負不合），

通分，

求值式 =  $\frac{(1+\sin\theta)+(1-\sin\theta)}{(1-\sin\theta)(1+\sin\theta)} = \frac{2}{1-\sin^2\theta} = \frac{2}{\cos^2\theta} = \frac{2}{\sin\theta} = \frac{2}{\frac{-1+\sqrt{5}}{2}} = \frac{4}{\sqrt{5}-1} = \sqrt{5} + 1$  .

31. 如圖，矩形  $ABCD$  中， $\overline{AB} = 12$ ， $\overline{BC} = 5$ ，若  $P$  為  $\overline{CD}$  上一點， $\angle PAD = \alpha$ ， $\angle PBC = \beta$ ，則  $\tan \alpha + \tan \beta =$  \_\_\_\_\_ .



**解答**  $\frac{12}{5}$

**解析**  $\tan \alpha + \tan \beta = \frac{\overline{DP}}{\overline{AD}} + \frac{\overline{CP}}{\overline{BC}} = \frac{\overline{DP}}{\overline{BC}} + \frac{\overline{CP}}{\overline{BC}} = \frac{\overline{DP} + \overline{CP}}{\overline{BC}} = \frac{\overline{AB}}{\overline{BC}} = \frac{12}{5}$  .