高雄市明誠中學 高一數學平時測驗 日期：92.04.09					
範圍	2-1&2(2)三角函數 +Ans	班級		姓名	
		座號			

一. 單一選擇題 (每題 10 分)

1、(C) 化簡$\dfrac{1}{4\sin^2\theta+2}+\dfrac{1}{\csc^2\theta+2}=$ (A)0 (B)$\dfrac{1}{4}$ (C)$\dfrac{1}{2}$ (D)1 (E)2

解析：$\dfrac{1}{4\sin^2\theta+2}+\dfrac{2\sin^2\theta}{2+4\sin^2\theta}=\dfrac{1}{2}$

2、(A) 求$\sin 23°\cos 67°-\cos 23°\csc 67°+\cos^2 23°$之值為

(A)0 (B)1 (C)2 (D)$\sin 23°\cos 67°$ (E)$\cos^2 23°$

解析：$\sin^2 23°-1+\cos^2 23°=0$

3、(A) 設$\angle A$為銳角，$(2\sec A-3)(7\sec A-4)=0$，則$\sec A=$ (A)$\dfrac{3}{2}$ (B)$\dfrac{4}{7}$ (C)$\dfrac{3}{2}$或$\dfrac{4}{7}$

(D)$\dfrac{3}{2}$且$\dfrac{4}{7}$ (E)無解

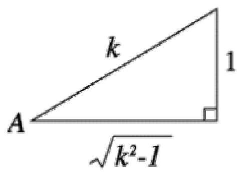
解析：$\because |\sec A|\ge 1$ $\therefore \sec A=\dfrac{3}{2}$

4、(B) 設$\angle A$為銳角且$\csc A=k$，則下列何者正確？

(A)$\sin A=\dfrac{k}{\sqrt{k^2+1}}$ (B)$\cos A=\dfrac{\sqrt{k^2-1}}{k}$ (C)$\tan A=\sqrt{k^2-1}$ (D)$\cot A=\dfrac{1}{\sqrt{k^2-1}}$

(E)$\sec A=\sqrt{k^2+1}$

解析：$\therefore \cos A=\dfrac{\sqrt{k^2-1}}{k}$



二. 填充題 (每題 10 分)

1、設$\sin\theta=\dfrac{2}{5}$，求$\dfrac{7\cos\theta-3\cot\theta}{4\cos\theta+6\cot\theta}=$____________。

答案：由$\sin\theta=\dfrac{\cos\theta}{\cot\theta}=\dfrac{2}{5}$，令$\cos\theta=2k,\ \cot\theta=5k$，則

$\dfrac{7\cos\theta-3\cot\theta}{4\cos\theta+6\cot\theta}=\dfrac{14k-15k}{8k+30k}=-\dfrac{1}{38}$

2、設$\sin\theta+\cos\theta=\dfrac{7}{5}$，且$0<\theta<45°$,則(1)$\sin\theta\cos\theta=$______, (2)$\sin\theta-\cos\theta=$______,

(3)$\tan\theta+\cot\theta=$______, (4)$\sin^3\theta+\cos^3\theta=$______ 。

(提示：$\because 0<\theta<45°$ $\therefore \sin\theta<\cos\theta$)

答案：(1)$\dfrac{12}{25}$ (2)$-\dfrac{1}{5}$ (3)$\dfrac{25}{12}$ (4)$\dfrac{91}{125}$

解析：$\sin\theta+\cos\theta=\dfrac{7}{5}$　$\therefore 2\sin\theta\cdot\cos\theta=\dfrac{49}{25}-1$　$\therefore \sin\theta\cdot\cos\theta=\dfrac{12}{25}$，$(\sin\theta-\cos\theta)^2=\dfrac{1}{25}$

$\because 0<\theta<45°$　$\therefore \sin\theta<\cos\theta$　$\therefore \sin\theta-\cos\theta=-\dfrac{1}{5}$

$\tan\theta+\cot\theta=\dfrac{1}{\sin\theta\cdot\cos\theta}=\dfrac{25}{12}$

$\sin^3\theta+\cos^3\theta=(\sin\theta+\cos\theta)(1-\sin\theta\cos\theta)=\dfrac{7}{5}\times\dfrac{13}{25}=\dfrac{91}{125}$

3、設$\sin\theta+\cos\theta=\dfrac{6}{5}$，則$\dfrac{\sin\theta}{1-\cot\theta}+\dfrac{\cos\theta}{1-\tan\theta}=$______。

答案：$\dfrac{6}{5}$

解析：$\dfrac{\sin\theta}{1-\cot\theta}+\dfrac{\cos\theta}{1-\tan\theta}=\dfrac{\sin^2\theta}{\sin\theta-\cos\theta}+\dfrac{\cos^2\theta}{\cos\theta-\sin\theta}=\sin\theta+\cos\theta=\dfrac{6}{5}$

4、設$0°<\angle A<18°$, $\sin A=\cos 5A$，則$\angle A=$______，又$\sec 4A=$______。

答案：$15°, 2$

解析：$\sin A=\cos 5A=\sin(90°-5A)$　$\therefore \angle A=15°$, $\sec 60°=2$

5、設$\triangle ABC$在XY平面上之坐標分別爲$A(1,-1)$，$B(-3,-4)$，$C(-9,4)$，則

(1) $\overline{BC}=$______, (2) $\cos A=$______。

答案：(1)10　(2)$\dfrac{\sqrt{5}}{5}$

解析：(1) $\overline{BC}=\sqrt{(-9+3)^2+(4+4)^2}=\sqrt{100}=10$

(2) $\overline{AC}=\sqrt{125}, \overline{AB}=5$　$\therefore \angle B=90°$　$\therefore \cos A=\dfrac{5}{\sqrt{125}}=\dfrac{\sqrt{5}}{5}$

6、設$\angle A$爲銳角，$2\cos^2 A+5\cos A-3=0$，則$\cos A=$_______, $\sin A+\sec A=$_______。

答案：$\dfrac{1}{2}$, $\dfrac{\sqrt{3}}{2}+2$

解析：$\because$(2cos$A$−1)(cos$A$+3)=0　$\therefore \cos A=\dfrac{1}{2}$或−3(不合)，故$\angle A=60°$

即$\sin A+\sec A=\dfrac{\sqrt{3}}{2}+2$

7、設$\angle A$爲銳角，若$x^2-4\cos Ax+1=0$有等根，則$\angle A=$______，又此等根爲$x=$______。

答案：$60°, 1$

解析：$D=0$　$\therefore \cos A=\pm\dfrac{1}{2}$　($-\dfrac{1}{2}$不合)　$\therefore \angle A=60°$　$x^2-2x+1=0$，等根爲$x=1$

8、設$\angle A$爲銳角，若$\sec^2\theta=3\tan\theta-1$，則$\sin A=$______或______。

答案：$\dfrac{\sqrt{2}}{2}$, $\dfrac{2}{\sqrt{5}}$

解析：令$\tan\theta=t$　$\therefore t^2+1=3t-1$　$\therefore t=1$或2　$\therefore \sin A=\dfrac{\sqrt{2}}{2}$或$\dfrac{2}{\sqrt{5}}$

9、若θ爲銳角，$4\sin^2\theta-4\cos^2\theta=2+\sin\theta\cos\theta$，則$\tan\theta=$______，又$\sin\theta=$______。(提示：同除$\cos^2\theta$)

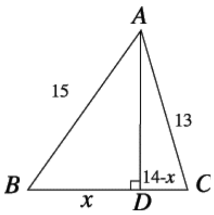
答案：2, $\frac{2\sqrt{5}}{5}$

解析：同除$\cos^2\theta \Rightarrow 4\tan^2\theta - 4 = 2\sec^2\theta + \tan\theta$　　$\therefore \tan\theta = 2$或$\tan\theta = -\frac{3}{2}$(不合)

$\therefore \sin\theta = \frac{2}{\sqrt{5}} = \frac{2\sqrt{5}}{5}$

10、$\triangle ABC$中，若$\overline{AB} = 15,\ \overline{AC} = 13,\ \overline{BC} = 14,\ \overline{AD} \perp \overline{BC}$於$D$，則$\overline{BD} =$_________，又$\csc C =$____________

答案：

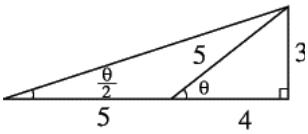


設$\overline{BD} = x, \overline{CD} = 14 - x$　$\therefore 15^2 - x^2 = 13^2 - (14 - x)^2$　$\therefore x = 9$

$\therefore \overline{CD} = 5, \overline{AD} = 12$　$\therefore \csc C = \frac{13}{12}$

11、已知$\sin\theta = \frac{3}{5}$，試利用幾何圖形求$\sin\frac{\theta}{2} =$_________與$\cos\frac{\theta}{2} =$__________。

答案：



$\sqrt{9^2 + 3^2} = 3\sqrt{10}$　　$\sin\frac{\theta}{2} = \frac{1}{\sqrt{10}}, \cos\frac{\theta}{2} = \frac{3}{\sqrt{10}}$