

高雄市明誠中學 高一數學平時測驗				日期：92.02.26	
範圍	1-1.指數函數+Ans	班級		姓名	
		座號			

一. 填充題 (每題 0 分)

1、若 $3^x = 13$, $13^y = 27$, 則 $x \cdot y =$ _____。

答案：3

解析： $(3^x)^y = 13^y = 27 \quad \therefore 3^{xy} = 27 \quad \therefore xy = 3$

2、設 $2^x = 3^y = 36$, 則 $\frac{2}{x} + \frac{2}{y} =$ _____。

答案：1

解析： $2^x = 3^y = 36 \quad \therefore 2 = 6^{\frac{2}{x}}, 3 = 6^{\frac{2}{y}} \quad \therefore 6^{\frac{2}{x} + \frac{2}{y}} = 2 \times 3 = 6 \quad \text{故 } \frac{2}{x} + \frac{2}{y} = 1$

3、 $x, y \in \mathbf{Z}$, $2^x \cdot 3^y = 54$, 則 (1) $2^{x-1} \cdot 3^{y+1} =$ _____, (2) $\frac{2^{x+1}}{3^{y-1}} =$ _____。

答案：(1) 81 (2) $\frac{4}{9}$

解析：(1) $\because x, y \in \mathbf{Z}, 2^x \times 3^y = 2^1 \times 3^3 \quad \therefore x=1, y=3 \quad \therefore 2^{x-1} \times 3^{y+1} = 3^4 = 81$

(2) $\frac{2^2}{3^2} = \frac{4}{9}$

4、設 $x > 0$, 若 $\sqrt[7]{x^{\frac{5}{3}}} \cdot \sqrt[5]{x^{\frac{3}{7}}} \cdot \sqrt[3]{x^{\frac{7}{5}}} = x^k$, 則 $k =$ _____。

答案： $\frac{83}{105}$

解析： $(x^{\frac{5}{3}})^{\frac{1}{7}} \cdot (x^{\frac{3}{7}})^{\frac{1}{5}} \cdot (x^{\frac{7}{5}})^{\frac{1}{3}} = x^{\frac{5}{21} + \frac{3}{35} + \frac{7}{15}} = x^{\frac{83}{105}}$

5、化簡 $\sqrt[3]{192} + \frac{4}{\sqrt[3]{-9}} + \frac{1}{3}\sqrt[6]{9} = 3^k$, 則 $k =$ _____。

答案： $\frac{4}{3}$

解析： $\sqrt[3]{3 \times 4^3} + \frac{4}{-3^{\frac{2}{3}}} + \frac{1}{3} \cdot 3^{\frac{1}{3}} = 4 \times 3^{\frac{1}{3}} - \frac{4}{3} \times 3^{\frac{1}{3}} + \frac{1}{3} \times 3^{\frac{1}{3}} = 3 \times 3^{\frac{1}{3}} = 3^{\frac{4}{3}} \quad \therefore k = \frac{4}{3}$

6、設 $a > 0$, $\frac{[a^3 \cdot (a^{-2})^2]^4}{(a\sqrt{a})^3} = a^k$, 則 $k =$ _____。

答案： $-\frac{17}{2}$

解析： $\frac{[a^{-1}]^4}{(a^{\frac{3}{2}})^3} = a^{-\frac{17}{2}} \quad \therefore k = -\frac{17}{2}$

7、化簡(1) $(\frac{1}{32})^{0.2} =$ _____, (2) $(243)^{0.1} =$ _____。

答案：(1) $\frac{1}{2}$ (2) $\sqrt{3}$

解析：(1) $(\frac{1}{32})^{0.2} = (2^{-5})^{0.2} = 2^{-1} = \frac{1}{2}$ (2) $(243)^{0.1} = (3^5)^{0.1} = 3^{\frac{1}{2}} = \sqrt{3}$

8、若 $e^{2x} = 2$ ，則 $\frac{3e^x + e^{-x}}{e^{3x} + e^x} = \underline{\hspace{2cm}}$ 。

答案： $\frac{7}{6}$

解析： $\frac{3e^x + e^{-x}}{e^{3x} + e^x} = \frac{3e^{2x} + 1}{e^{4x} + e^{2x}} = \frac{7}{6}$

9、化簡(1) $\sqrt[4]{(\sqrt[3]{a^5})^9} = \underline{\hspace{2cm}}$ ，(2) $\sqrt{\sqrt{a^2b^3} \cdot \sqrt[3]{a^4b^3}} = \underline{\hspace{2cm}}$ 。

答案：(1) $a^3\sqrt[4]{a^3}$ (2) $a^{\frac{7}{6}} \cdot b^{\frac{5}{4}}$

解析：(1) $\sqrt[4]{(\sqrt[3]{a^5})^9} = \sqrt[4]{(a^{\frac{5}{3}})^9} = a^{\frac{15}{4}} = a^3\sqrt[4]{a^3}$

(2) $\sqrt{\sqrt{a^2b^3} \cdot \sqrt[3]{a^4b^3}} = \sqrt{ab^{\frac{3}{2}} \cdot a^{\frac{4}{3}} \cdot b} = (a^{\frac{7}{3}}b^{\frac{5}{2}})^{\frac{1}{2}} = a^{\frac{7}{6}} \cdot b^{\frac{5}{4}}$

10、設 $x > 0$ ，(1)若 $\sqrt{\frac{x^5}{x^3}} = x^k$ ，則 $k = \underline{\hspace{2cm}}$ 。(2)若 $\sqrt{\frac{x^5}{x^3}} \cdot \sqrt[3]{\frac{x^2}{x^5}} \cdot \sqrt[5]{\frac{x^3}{x^2}} = x^\ell$ ，則 $\ell = \underline{\hspace{2cm}}$ 。

答案：(1) 1 (2) $\frac{1}{5}$

解析：(1) $\sqrt{\frac{x^5}{x^3}} = \sqrt{x^2} = x \quad \therefore k = 1$

(2) $\sqrt{\frac{x^5}{x^3}} \cdot \sqrt[3]{\frac{x^2}{x^5}} \cdot \sqrt[5]{\frac{x^3}{x^2}} = x \cdot (x^{-3})^{\frac{1}{3}} \cdot (x)^{\frac{1}{5}} = x^{\frac{1}{5}} \quad \therefore \ell = \frac{1}{5}$

11、 $\sqrt[3]{5^{18}} \times \sqrt{\sqrt{5^8}} \times [(\frac{1}{25})^2 \times (125)^2]^{-3} = 5^k$ ，則 $k = \underline{\hspace{2cm}}$ 。

答案：2

解析：原式 $= 5^6 \times 5^2 \times (5^{-4} \times 5^6)^{-3} = 5^2$ ， $\therefore k = 2$

12、設 $13^x = 8$ ， $52^y = 16$ ，則(1) $2^{\frac{3}{x}} = \underline{\hspace{2cm}}$ ，(2) $\frac{6}{x} - \frac{8}{y} = \underline{\hspace{2cm}}$ 。

答案：(1) 13 (2) -4

解析：(1) $13^x = 2^3 \quad \therefore 2^{\frac{3}{x}} = 13$

(2) $52^y = 16 \quad \therefore 2^{\frac{4}{y}} = 52 \quad \therefore 2^{\frac{3-4}{x-y}} = \frac{1}{4} \quad \therefore \frac{3}{x} - \frac{4}{y} = -2 \quad \therefore \frac{6}{x} - \frac{8}{y} = -4$

13、設 $2^{3^2} \div (2^3)^2 = 2^k$ ，則 $k = \underline{\hspace{2cm}}$ 。

答案：3

解析： $2^{3^2} \div (2^3)^2 = 2^9 \div 2^6 = 2^3 \quad \therefore k = 3$

14、(1)化簡 $[(\frac{27}{64})^{\frac{1}{4}}]^{\frac{2}{3}} = \underline{\hspace{2cm}}$ 。(2)求 $(\frac{8}{27})^{-\frac{1}{3}} \cdot (\frac{1}{16})^{\frac{1}{4}} \cdot (0.25)^{-2.5}$ 之值 $= \underline{\hspace{2cm}}$ 。

答案：(1) $\frac{2\sqrt{3}}{3}$ (2) 24

解析：(1) $\left[\left(\frac{27}{64}\right)^{\frac{1}{4}}\right]^{\frac{2}{3}} = \left[\left(\left(\frac{3}{4}\right)^{\frac{3}{4}}\right)^{\frac{2}{3}}\right] = \left(\frac{3}{4}\right)^{\frac{1}{2}} = \frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$

(2) $\left(\frac{8}{27}\right)^{\frac{1}{3}} \cdot \left(\frac{1}{16}\right)^{\frac{1}{4}} \cdot (0.25)^{-2.5} = \frac{3}{2} \times \frac{1}{2} \times 32 = 24$

15、若 $2^x + 3^y = 7$, $2^{x-1} + 3^{y+1} = 16$, 則 $2^{x+1} + 3^{y-1} =$ _____。

答案： $\frac{17}{3}$

解析：令 $A = 2^x$, $B = 3^y$ $\therefore A + B = 7$, $\frac{A}{2} + 3B = 16$

得 $B = 5$, $A = 2$ 故 $2^{x+1} + 3^{y-1} = 2A + \frac{B}{3} = \frac{17}{3}$

16、設 $x^{\frac{1}{2}} - x^{-\frac{1}{2}} = 2$, 則 $x + x^{-1} =$ _____, $x^{\frac{1}{2}} + x^{-\frac{1}{2}} =$ _____。

答案： $6, 2\sqrt{2}$

解析： $x^{\frac{1}{2}} - x^{-\frac{1}{2}} = 2$ $\therefore x - 2 + \frac{1}{x} = 4$ $\therefore x + x^{-1} = 6$

$x + 2 + \frac{1}{x} = 8$ $\therefore x^{\frac{1}{2}} + x^{-\frac{1}{2}} = 2\sqrt{2}$ ($-2\sqrt{2}$ 不合)

二. 計算與證明題 (每題 10 分)

1、設 $5^x + 5^{-x} = 4$, 則 $5^{2x} + 5^{-2x} = ?$, 又 $5^x - 5^{-x} = ?$

答案： $5^x + 5^{-x} = 4$, $5^{2x} + 2 + 5^{-2x} = 16$ $\therefore 5^{2x} + 5^{-2x} = 14$

$5^{2x} - 2 + 5^{-2x} = 12$ $\therefore 5^x - 5^{-x} = \pm 2\sqrt{3}$

2、設 $\sqrt[4]{16} = \sqrt[2]{2^{y+3}}$ 且 $3^{2y+6x} = 27^{xy}$, 求 x, y 之值。

答案： $2^{\frac{4}{x}} = 2^{\frac{y+3}{y}}$ 且 $3^{2y+6x} = 3^{3xy}$ $\therefore \frac{4}{x} = \frac{3}{y} + 1$ 且 $\frac{2y+6x}{xy} = 3$ $\therefore x = 2$, $y = 3$

3、設 $4^{2x} = 9$, 則(1) $4^x = ?$ (2) $2^{x+1} = ?$ (3) $\frac{2^x - 2^{-x}}{2^{3x} + 2^{-x}} = ?$

答案：(1) $4^{2x} = 9$ $\therefore 4^x = 3$ (-3 不合)

(2) $2^x = \sqrt{3}$ ($-\sqrt{3}$ 不合) $\therefore 2^{x+1} = 2\sqrt{3}$

(3) $\frac{2^x - 2^{-x}}{2^{3x} + 2^{-x}} = \frac{2^{2x} - 1}{2^{4x} + 1} = \frac{2}{10} = \frac{1}{5}$