高雄市明誠中學 高一數學平時測驗 日期：93.05.27					
範圍	3-2 和角公式+Ans	班級		姓名	
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一. 選擇題(每題 10 分)

1. 在△ABC 中，已知 $\tan A$．$\tan B = 1$，則下列何者恆正確？

(A) $\overline{AB}=\overline{AC}$ (B) $\angle C = 90°$ (C)$\angle A = 45°$ (D) $\overline{AB}=\overline{BC}$ (E)$\angle A=\angle B$

答案：(B)

解析：$\tan A$．$\tan B = 1 \Rightarrow \dfrac{\sin A}{\cos A}$．$\dfrac{\sin B}{\cos B} = 1 \Rightarrow \sin A\sin B = \cos A\cos B$

$\Rightarrow \cos A\cos B - \sin A\sin B = 0 \Rightarrow \cos(A+B) = 0$

$\Rightarrow \cos(\pi - C) = 0$（$\because$ $A+B+C=\pi \Rightarrow A+B=\pi - C$） $\Rightarrow \cos C = 0$

$\therefore$ $\angle C = 90°$，故選(B)

2.下列敘述，何者正確？(複選)

(A) $\sin(\alpha+\beta)\sin(\alpha-\beta)=\cos^2\beta-\cos^2\alpha$ (B) $\cos(\alpha+\beta)\cos(\alpha-\beta)=\sin^2\beta-\cos^2\alpha$

(C) $\cos(\alpha+\beta)\cos(\alpha-\beta)=\cos^2\beta-\sin^2\alpha$ (D) $\sin(60°-\theta)\sin\theta\sin(60°+\theta)=\dfrac{1}{4}\sin3\theta$

(E) $\cos(60°-\theta)\cos\theta\cos(60°+\theta)=\dfrac{1}{4}\cos3\theta$。

答案：(A)(C)(D)(E)

解析：(1) $\sin(\alpha+\beta)\sin(\alpha-\beta)=(\sin\alpha\cos\beta+\cos\alpha\sin\beta)(\sin\alpha\cos\beta-\cos\alpha\sin\beta)$

$=\sin^2\alpha\cos^2\beta-\cos^2\alpha\sin^2\beta \quad =\sin^2\alpha(1-\sin^2\beta)-(1-\sin^2\alpha)\sin^2\beta$

$=\sin^2\alpha-\sin^2\beta=\cos^2\beta-\cos^2\alpha$

(2) $\cos(\alpha+\beta)\cos(\alpha-\beta)=(\cos\alpha\cos\beta-\sin\alpha\sin\beta)(\cos\alpha\cos\beta+\sin\alpha\sin\beta)$

$=\cos^2\alpha\cos^2\beta-\sin^2\alpha\sin^2\beta \quad =\cos^2\alpha(1-\sin^2\beta)-(1-\cos^2\alpha)\sin^2\beta$

$=\cos^2\alpha-\sin^2\beta=\cos^2\beta-\sin^2\alpha$

(3) $\sin(60°-\theta)\sin\theta\sin(60°+\theta)$

$=(\sin^2 60°-\sin^2\theta)\sin\theta=(\dfrac{3}{4}-\sin^2\theta)\sin\theta=\dfrac{1}{4}(3\sin\theta-4\sin^3\theta)=\dfrac{1}{4}\sin3\theta$

(4) $\cos(60°-\theta)\cos\theta\cos(60°+\theta)=(\cos^2 60°-\sin^2\theta)\cos\theta$

$=(\dfrac{1}{4}-1+\cos^2\theta)\cos\theta=\cos^3\theta-\dfrac{3}{4}\cos\theta=\dfrac{1}{4}(4\cos^3\theta-3\cos\theta)=\dfrac{1}{4}\cos3\theta$

二、填充題(每題 10 分)

3.已知$\tan\theta = 2$，則$\tan(\dfrac{\pi}{4}+\dfrac{\theta}{2})-\tan(\dfrac{\pi}{4}-\dfrac{\theta}{2})$之值爲 __________ 。

答案：4

解析：$\tan(\dfrac{\pi}{4}+\dfrac{\theta}{2})-\tan(\dfrac{\pi}{4}-\dfrac{\theta}{2})=\dfrac{\tan\frac{\pi}{4}+\tan\frac{\theta}{2}}{1-\tan\frac{\pi}{4}\tan\frac{\theta}{2}}-\dfrac{\tan\frac{\pi}{4}-\tan\frac{\theta}{2}}{1+\tan\frac{\pi}{4}\tan\frac{\theta}{2}}$

$$=\frac{1+\tan\frac{\theta}{2}}{1-\tan\frac{\theta}{2}}-\frac{1-\tan\frac{\theta}{2}}{1+\tan\frac{\theta}{2}}=\frac{(1+\tan\frac{\theta}{2})^2-(1-\tan\frac{\theta}{2})^2}{1-\tan^2\frac{\theta}{2}}=\frac{4\tan\frac{\theta}{2}}{1-\tan^2\frac{\theta}{2}}$$

$$=2\tan(\frac{\theta}{2}+\frac{\theta}{2})=2\tan\theta=2\times2=4$$

4. 試求$\sin23°\cos112°-\sin292°\sin67°=$＿＿＿＿＿。

答案：$\frac{\sqrt{2}}{2}$

解析：原式 $=-\sin23°\cos68°+\sin68°\cos23°=\sin(68°-23°)=\sin45°=\frac{\sqrt{2}}{2}$

5. 設$\pi<\alpha<\frac{3\pi}{2}$，$-\frac{\pi}{2}<\beta<0$ 且$\tan\alpha=\frac{1}{2}$，$\cot\beta=-3$，則$\alpha-\beta=$＿＿＿弧度。

答案：$\frac{5\pi}{4}$

解析：$\pi<\alpha<\frac{3\pi}{2}$，$-\frac{\pi}{2}<\beta<0 \quad\Rightarrow\quad \pi<\alpha-\beta<2\pi$

$$\tan(\alpha-\beta)=\frac{\tan\alpha-\tan\beta}{1+\tan\alpha\tan\beta}=\frac{\frac{1}{2}-(\frac{-1}{3})}{1+\frac{1}{2}(\frac{-1}{3})}=1 \quad \therefore \quad \alpha-\beta=\frac{5\pi}{4}$$

6. 設 $0<\alpha<\frac{\pi}{2}<\beta<\pi$，且$\sin\alpha=\frac{13}{14}$，$\sin\beta=\frac{11}{14}$，則

(1) $\cos(\alpha-\beta)=$＿＿＿＿＿。 (2) $\alpha-\beta=$＿＿＿＿＿。

答案：(1)$\frac{1}{2}$ (2)$-\frac{\pi}{3}$

解析：$\because\ 0<\alpha<\frac{\pi}{2}<\beta<\pi$且 $\sin\alpha=\frac{13}{14}$，$\sin\beta=\frac{11}{14}$ $\quad\therefore\quad \cos\alpha=\frac{3\sqrt{3}}{14}$，$\cos\beta=-\frac{5\sqrt{3}}{14}$

故 $\cos(\alpha-\beta)=\cos\alpha\cos\beta+\sin\alpha\sin\beta=\frac{3\sqrt{3}}{14}(-\frac{5\sqrt{3}}{14})+\frac{13}{14}\cdot\frac{11}{14}=\frac{98}{196}=\frac{1}{2}$

$\because\ -\pi<-\beta<-\frac{\pi}{2}$且 $0<\alpha<\frac{\pi}{2}$ $\quad\therefore\quad -\pi<\alpha-\beta<0$，故$\alpha-\beta=-\frac{\pi}{3}$

7. 設$\frac{\pi}{2}<\alpha<\pi$，$\pi<\beta<\frac{3}{2}\pi$，且$\sin\alpha=\frac{3}{5}$，$\cos\beta=-\frac{1}{4}$，則$\cos(\alpha+\beta)=$＿＿＿＿＿。

答案：$\frac{4+3\sqrt{15}}{20}$

解析：$\because\ \frac{\pi}{2}<\alpha<\pi \quad\therefore\quad \sin\alpha=\frac{3}{5}$，畫圖$\Rightarrow\cos\alpha=-\frac{4}{5}$

$\because\ \pi<\beta<\frac{3}{2}\pi \quad\therefore\quad \cos\beta=-\frac{1}{4}$，畫圖$\Rightarrow\sin\beta=-\frac{\sqrt{15}}{4}$

$$\cos(\alpha+\beta)=\cos\alpha\cos\beta-\sin\alpha\sin\beta=\frac{-4}{5}\times\frac{-1}{4}-\frac{3}{5}\times\frac{-\sqrt{15}}{4}=\frac{4+3\sqrt{15}}{20}$$

8.設 $0<\alpha<\frac{\pi}{2}$，$\pi<\beta<\frac{3\pi}{2}$，$\tan\alpha=\frac{1}{2}$，$\tan\beta=\frac{1}{3}$，則

(1) $\tan(\alpha+\beta)=$ __________ 。 (2) $\alpha+\beta=$ __________ 。

答案：(1) 1 (2) $\frac{5\pi}{4}$

解析：(1) $\tan(\alpha+\beta)=\frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}=\frac{\frac{1}{2}+\frac{1}{3}}{1-\frac{1}{2}\cdot\frac{1}{3}}=1$ (2)$\because$ $\pi<\alpha+\beta<2\pi$ $\therefore$ $\alpha+\beta=\frac{5\pi}{4}$

9. 求$\sqrt{3}\tan80°\tan20°-\tan80°+\tan20°$的值爲__________ 。

答案：$-\sqrt{3}$

解析：$\tan(80°-20°)=\frac{\tan80°-\tan20°}{1+\tan80°\tan20°}=\tan60°=\sqrt{3}$

$\Rightarrow$ $\sqrt{3}+\sqrt{3}\tan80°\tan20°=\tan80°-\tan20°$

$\therefore$ $\sqrt{3}\tan80°\tan20°-\tan80°+\tan20°=-\sqrt{3}$

10.$\cos^2\frac{\pi}{24}-\sin^2\frac{5\pi}{24}=$ __________ 。

答案：$\frac{\sqrt{6}}{4}$

解析：$\cos^2\frac{\pi}{24}-\sin^2\frac{5\pi}{24}=\cos(\frac{\pi}{24}+\frac{5\pi}{24})\cos(\frac{\pi}{24}-\frac{5\pi}{24})=\cos\frac{\pi}{4}\cos\frac{\pi}{6}=\frac{\sqrt{2}}{2}\times\frac{\sqrt{3}}{2}=\frac{\sqrt{6}}{4}$

11.$\cos40°\sin160°-\sin220°\cos340°=$ __________ ，而$(1+\tan35°)(1+\tan10°)=$ __________ 。

答案：$\frac{\sqrt{3}}{2}$；2

解析：利用和角公式

(1) $\cos40°\sin160°-\sin220°\cos340°=\cos40°\sin20°+\sin40°\cos20°$

$=\sin20°\cos40°+\cos20°\sin40°=\sin(20°+40°)=\sin60°=\frac{\sqrt{3}}{2}$

(2)$\because$ $35°+10°=45°$ $\therefore$ $\tan(35°+10°)=\tan45°$ $\Rightarrow$ $\frac{\tan35°+\tan10°}{1-\tan35°\tan10°}=1$

$\Rightarrow$ $\tan35°+\tan10°=1-\tan35°\tan10°$ $\Rightarrow$ $\tan35°\cdot\tan10°+\tan35°+\tan10°=1$

$\therefore$ $(1+\tan35°)(1+\tan10°)=1+(\tan35°\cdot\tan10°+\tan35°+\tan10°)=1+1=2$

12.$P(\cos\alpha，\sin\alpha)$，$Q(\cos\beta，\sin\beta)$，$\alpha-\beta=\frac{3}{4}\pi$，求$P$到$Q$之距離 = __________ 。

答案：$\sqrt{2+\sqrt{2}}$

解析：$\overline{PQ}=\sqrt{(\cos\alpha-\cos\beta)^2+(\sin\alpha-\sin\beta)^2}$

$=\sqrt{\cos^2\alpha-2\cos\alpha\cos\beta+\cos^2\beta+\sin^2\alpha-2\sin\alpha\sin\beta+\sin^2\beta}$

$=\sqrt{2-2(\cos\alpha\cos\beta+\sin\alpha\sin\beta)}=\sqrt{2-2\cos(\alpha-\beta)}$

$=\sqrt{2-2\cos\frac{3\pi}{4}}=\sqrt{2+2\cos\frac{\pi}{4}}=\sqrt{2+\sqrt{2}}$

13.設$\sin84°=a$，$\cos63°=b$，若以a，b表$\cos21°$及$\sin147°$，則$\cos21°=$ __________ ，而$\sin147°$ = __________ 。

答案：$b\sqrt{1-a^2}+a\sqrt{1-b^2}$；$ab+\sqrt{1-a^2}\ \sqrt{1-b^2}$

解析：∵ $\sin 84° = a$，$\cos 63° = b$ ∴ $\cos 84° = \sqrt{1-a^2}$，$\sin 63° = \sqrt{1-b^2}$

(1) $\cos 21° = \cos(84° - 63°) = \cos 84° \cos 63° + \sin 84° \sin 63°$

$= \sqrt{1-a^2} \cdot b + a\sqrt{1-b^2} = b\sqrt{1-a^2} + a\sqrt{1-b^2}$

(2) $\sin 147° = \sin(84° + 63°) = \sin 84° \cos 63° + \cos 84° \sin 63° = a \cdot b + \sqrt{1-a^2}\ \sqrt{1-b^2}$

14.設$\tan\alpha$，$\tan\beta$ 爲$x^2 + 6x + 3 = 0$ 之二根，則

(1) $\tan(\alpha+\beta) =$ ＿＿＿＿＿＿。 (2) $\cos^2(\alpha+\beta) =$＿＿＿＿＿＿。

(3) $\sin^2(\alpha+\beta) + 2\sin(\alpha+\beta)\cos(\alpha+\beta) + 5\cos^2(\alpha+\beta)$之值爲＿＿＿＿＿＿。

答案：(1) 3 (2) $\frac{1}{10}$ (3) 2

解析：(1)由根與係數關係得$\tan\alpha + \tan\beta = -6$，$\tan\alpha\tan\beta = 3$

$\Rightarrow \tan(\alpha+\beta) = \dfrac{\tan\alpha + \tan\beta}{1-\tan\alpha\tan\beta} = \dfrac{-6}{1-3} = 3$

∴ $\cos^2(\alpha+\beta) = \dfrac{1}{\sec^2(\alpha+\beta)} = \dfrac{1}{1+\tan^2(\alpha+\beta)} = \dfrac{1}{1+3^2} = \dfrac{1}{10}$

(2)原式$= \cos^2(\alpha+\beta)\left[\dfrac{\sin^2(\alpha+\beta)}{\cos^2(\alpha+\beta)} + \dfrac{2\sin(\alpha+\beta)\cos(\alpha+\beta)}{\cos^2(\alpha+\beta)} + \dfrac{5\cos^2(\alpha+\beta)}{\cos^2(\alpha+\beta)}\right]$

$= \cos^2(\alpha+\beta)[\tan^2(\alpha+\beta) + 2\tan(\alpha+\beta) + 5] = \dfrac{1}{10}(3^2 + 2 \cdot 3 + 5) = 2$

15.$\cos\alpha - \cos\beta = \frac{1}{3}$，$\sin\alpha + \sin\beta = \frac{1}{2}$，求$\sin(\alpha-\beta) =$ ＿＿＿＿＿＿。

答案：$-\frac{12}{13}$

解析：$(\cos\alpha - \cos\beta)^2 = \cos^2\alpha + \cos^2\beta - 2\cos\alpha\cos\beta = \frac{1}{9}$ ……①

$(\sin\alpha + \sin\beta)^2 = \sin^2\alpha + \sin^2\beta + 2\sin\alpha\sin\beta = \frac{1}{4}$ ……②

① + ② $\Rightarrow$ $2 - 2(\cos\alpha\cos\beta - \sin\alpha\sin\beta) = \dfrac{13}{36}$

$\Rightarrow$ $2\cos(\alpha+\beta) = 2 - \dfrac{13}{36} = \dfrac{59}{36}$ $\Rightarrow$ $\cos(\alpha+\beta) = \dfrac{59}{72}$

$(\cos\alpha - \cos\beta)(\sin\alpha + \sin\beta) = \cos\alpha\sin\alpha + \cos\alpha\sin\beta - \sin\alpha\cos\beta - \cos\beta\sin\beta$

$= \dfrac{1}{2}\sin 2\alpha - \dfrac{1}{2}\sin 2\beta - \sin(\alpha-\beta) = \dfrac{1}{2}(\sin 2\alpha - \sin 2\beta) - \sin(\alpha-\beta)$

$= \dfrac{1}{2}[2\cos(\alpha+\beta)\sin(\alpha-\beta)] - \sin(\alpha-\beta) = \dfrac{1}{6}$

$\Rightarrow$ $\dfrac{1}{6} = \dfrac{59}{72}\sin(\alpha-\beta) - \sin(\alpha-\beta) = -\dfrac{13}{72}\sin(\alpha-\beta)$

∴ $\sin(\alpha-\beta) = -\dfrac{12}{13}$

16.設$\alpha + \beta = \frac{\pi}{6}$，則$(\sin\alpha + \sin\beta)(\sin\alpha - \sin\beta)$之最小值爲＿＿＿＿＿＿。

答案：$-\frac{1}{2}$

解析：$(\sin\alpha+\sin\beta)(\sin\alpha-\sin\beta)=\sin^2\alpha-\sin^2\beta$

$=\sin(\alpha+\beta)\sin(\alpha-\beta)=\sin\frac{\pi}{6}\sin(\alpha-\beta)=\frac{1}{2}\sin(\alpha-\beta)$

17.設$\alpha+\beta=45°$時，可得：$(1+\tan\alpha)(1+\tan\beta)=2$。求

$(1+\tan1°)(1+\tan2°)(1+\tan3°)(1+\tan4°)\cdots(1+\tan44°)(1+\tan45°)$之值。

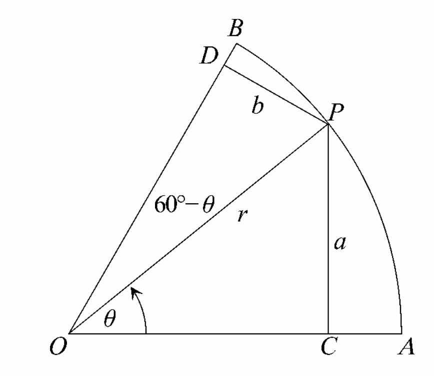
答案：2^{23}

解析：原式

$=[(1+\tan1°)(1+\tan44°)][(1+\tan2°)(1+\tan43°)]\cdots[(1+\tan22°)(1+\tan23°)](1+\tan45°)$

$=\overbrace{2\cdot2\cdots\cdots2}^{22個}\cdot(1+1)=2^{23}$

18.在扇形 OAB 中，O 爲圓心，$\overline{OA}=\overline{OB}=r$ 爲半徑，$\angle AOB=60°$。若 P 爲圓弧 $\widehat{AB}$ 上一點，而 P 至$\overline{OA}$的距離爲 a，P 至$\overline{OB}$的距離爲 b，試將 r 以 a，b 表示之。



答案：$r=\frac{2}{\sqrt{3}}\sqrt{a^2+ab+b^2}$

解析：

連結$\overline{OP}$，則$\overline{OP}=r$；再令$\angle AOP=\theta$，則$\angle BOP=60°-\theta$。

在△OPC中，$\sin\theta=\frac{a}{r}\Rightarrow a=r\sin\theta$ ……①

在△OPD中，$\sin(60°-\theta)=\frac{b}{r}\Rightarrow b=r\sin(60°-\theta)=r\sin60°\cos\theta-r\cos60°\sin\theta$

$=\frac{\sqrt{3}}{2}r\cos\theta-\frac{1}{2}r\sin\theta=\frac{\sqrt{3}}{2}r\cos\theta-\frac{1}{2}a$

$\therefore\ r\cos\theta=\frac{2}{\sqrt{3}}(\frac{1}{2}a+b)$ ……②

故將①^2 + ②^{2}相加得 $r^2(\sin^2\theta+\cos^2\theta)=a^2+\frac{4}{3}(\frac{1}{4}a^2+ab+b^2)=\frac{4}{3}(a^2+ab+b^2)$

$\therefore\ r=\frac{2}{\sqrt{3}}\sqrt{a^2+ab+b^2}$